MATHEMATICAL DESCRIPTION OF THE GRAIN STORAGE AND DRYING MODEL

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The STOREDRY Grain Drying Model has been developed in Silsoe Research Institute. The model can be used for both drying and storage of grains. The heart of the model is the solution of the four partial differential equations. They describe conventional heat and moisture transfer in ventilated granular beds. The model includes two procedures of equation solution. The first procedure is based on the equilibrium idea. The essence of the equilibrium idea is the assumption that, because low temperature grain drying is a slow process taking several days, the grain can be regarded as reaching equilibrium (as to temperature and moisture content) with the air during each time step of the integration of the differential equations. The second procedure incorporates more sophisticated Nellist's solution of the equations in finite difference form. The comparison of the two methods shows that there are minor differences as to moisture content and temperature profiles. Program is written in Microsoft FORTRAN, version 5.1.

near ambient grain drying; modelling; deep bed; grain storage

GENERAL MATHEMATICAL DESCRIPTION OF THE DRYING AND STORING MODEL

The development of the four partial differential equations commonly used to describe conventional heat and moisture transfer in ventilated granular beds has been set out by S h a r p (1981). The equations are:

Moisture (mass) balance

$$G \frac{\partial H}{\partial x} = -\rho_g \frac{\partial M}{\partial t} - \varepsilon \rho_a \frac{\partial H}{\partial t} \quad (1)$$

Heat balance

$$G (c_a + c_v H) \frac{\partial T}{\partial x} = -\rho_g c_v (T - \Theta) \frac{\partial M}{\partial t} - hS (T - \Theta) - \varepsilon \rho_a (c_a + c_v H) \frac{\partial T}{\partial t} \quad (2)$$

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Moisture (mass) transfer

$$\frac{\partial M}{\partial t} = -k\,(M - M_e) \tag{3}$$

Heat transfer

$$\rho_g\,(c_g + c_w M)\,\frac{\partial \Theta}{\partial t} + hS\,(T - \Theta) + l_g \rho_g\,\frac{\partial M}{\partial t} \tag{4}$$

Spencer (1972) and Bakker-Arkema et al.(1967) have shown that the terms involving the partial differential $\partial T/\partial t$ and $\partial H/\partial t$ are small and can be neglected so that equations (1), (2), (3) and (4) become equations (5), (6), (7) and (8)

$$G\,\frac{\partial H}{\partial x} = -\rho_g\,\frac{\partial M}{\partial t} \tag{5}$$

$$G\,(c_a + c_v H)\,\frac{\partial T}{\partial x} = -\rho_g c_v\,(T - \Theta)\,\frac{\partial M}{\partial t} - hS\,(T - \Theta) \tag{6}$$

$$\frac{\partial M}{\partial t} = -k\,(M - M_e) \tag{7}$$

$$\rho_g\,(c_g + c_w M)\,\frac{\partial \Theta}{\partial t} + hS\,(T - \Theta) + l_g \rho_g\,\frac{\partial M}{\partial t} \tag{8}$$

EQUILIBRIUM MODEL

The equilibrium model used within STOREDRY was originally invented by Morey et al. (1979) but was programmed and developed for use in drying wheat or barley under UK conditions by Sharp (1982). The essence of the model is that low temperature grain drying is a slow process taking several days. The grain can be regarded as reaching equilibrium (as to temperature and moisture content) with the air during each time step of the integration of the differential equations describing the heat and mass balance and transfer within an elemental depth of the bed.

Letting

$$T_{in} = T\,(x, t)$$

$$T_{ex} = T\,(x + \Delta x, t)$$

$$\Theta_{in} = \Theta\,(x, t)$$

$$\Theta_{ex} = \Theta\,(x, t + \Delta t)$$

$$H_{in} = H\,(x, t)$$

$$H_{ex} = H\,(x + \Delta x, t)$$

$$M_{in} = M\,(x, t)$$

$$M_{ex} = M\,(x, t + \Delta t)$$

now the equilibrium assumption implies that

$$\Theta_{ex} = T_{ex}$$

$$erh\,(M_{ex}, \Theta_{ex}) = rh\,(H_{ex}, T_{ex})$$

i.e.

$$\Theta\,(x, t + \Delta t) = T\,(x + \Delta x, t) \tag{9}$$

$$erh\,(M_{ex}, \Theta_{ex}) = rh\,(H_{ex}, T_{ex}) \tag{10}$$

Adding equations (6) and (8) by eliminating the heat transfer term $hS\,(T-\Theta)$ and substituting $\partial H/\partial x$ for $\partial M/\partial t$ gives

$$G\,(c_a + c_v H)\,\frac{\partial T}{\partial x} + \rho_g\,(c_g + c_w M)\,\frac{\partial \Theta}{\partial t} + G\,(c_v\,(T - \Theta) + l_g)\,\frac{\partial H}{\partial x} = 0 \tag{11}$$

Equations (5), (9), (10) and (11) can be written in differential forms

$$G\,\frac{H_{ex} - H_{in}}{\Delta x} = -\rho_g\,\frac{M_{ex} - M_{in}}{\Delta t} \tag{12}$$

$$\Theta_{ex} = T_{ex} \tag{13}$$

$$erh\,(M_{ex}, T_{ex}) = rh\,(H_{ex}, T_{ex}) \tag{14}$$

$$G\,(c_a + c_v H_{in})\,\frac{T_{ex} - T_{in}}{\Delta x} + \rho_g\,(c_g + c_w M_{in})\,\frac{T_{ex} - \Theta_{in}}{\Delta t} + \tag{15}$$

$$+ G\,(c_v\,(T_{ex} - \Theta_{in}) + l_g)\,\frac{H_{ex} - H_{in}}{\Delta x} = 0$$

Equation (15) can be solved to find T_{ex}

$$T_{ex} = \frac{(c_g + c_w M_{in})\,R\Theta_{in} + (c_a + c_v H_{in})\,T_{in} - (H_{ex} - H_{in})\,(l_g - c_v \Theta_{in})}{(c_a + c_v H_{ex}) + (c_g + c_w M_{in})\,R} \tag{16}$$

where

$$R = \frac{\rho_g\,\Delta x}{G\,\Delta t} \tag{17}$$

Hence given M_{in}, Θ_{in}, H_{in}, T_{in}, the set of equations (5), (9), (10) and (11) may be solved simultaneously for the exhaust air conditions H_{ex}, T_{ex} from the elemental layer Δx and for the state of the grain M_{ex}, Θ_{ex} at the end of the time step Δt. The model incorporates equations for relative humidity under adsorption as well as desorption and a routine to prevent over-prediction of drying under non-equilibrium conditions. At each depth and time step, the change in crop moisture content predicted by the equilibrium solution is compared with that predicted by the thin-layer drying equation (3) and the smaller value is used.

NELLIST'S FOUR EQUATIONS' MODEL

N e l l i s t (1974, 1987) have found the solution of equations (5), (6), (7) and (8) in finite difference form

$$\Delta M = \frac{-k\,(M - M_e)\,\Delta t}{(1 + \frac{1}{2}k\,\Delta t)} \qquad (18)$$

$$\Delta\Theta = \frac{A + \frac{\rho_g\,\Delta M}{\Delta t}\left(\frac{2Y}{hS} + \frac{F\,\Delta x}{GE}\right)}{1 + \frac{\rho_g}{\Delta t}\left(\frac{2B}{hS} + \frac{\Delta z}{GE}\right)(B + c_w\,\Delta M)} \qquad (19)$$

$$\Delta\Theta = -\frac{\rho_g\,\Delta x}{GE\,\Delta t}\,[\Delta T\,(B + c_w\,\Delta M) - F\,\Delta M] \qquad (20)$$

$$\Delta H = -\frac{\rho_g\,\Delta x\,\Delta M}{G\,\Delta t} \qquad (21)$$

where

$$A = 2\,(T - \Theta) \qquad (22)$$

$$B = c_g + c_w M \qquad (23)$$

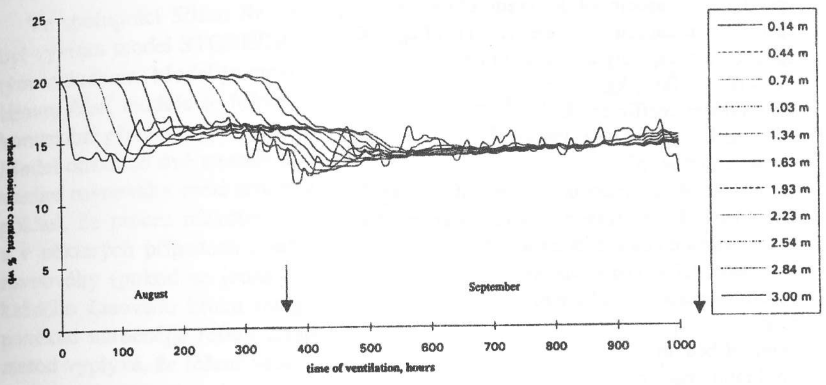
$$C = c_a + c_v\left(H - \frac{\rho_g\,\Delta x\,\Delta M}{G\,\Delta t}\right) \qquad (24)$$

$$D = l + c_v T - c_w \Theta$$

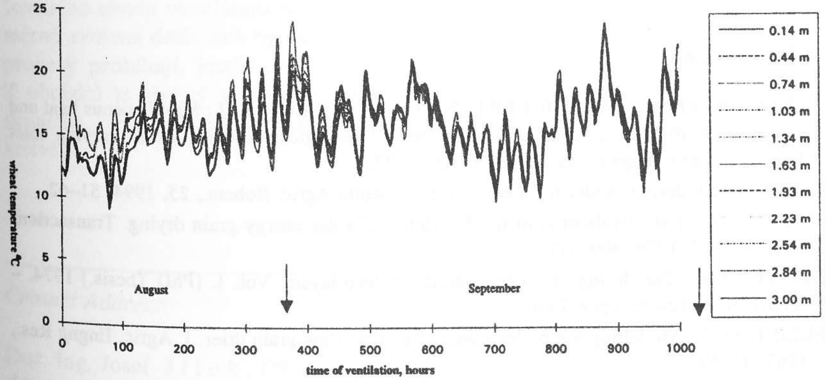
$$E = l_g + c_v T - c_w \Theta$$

EXAMPLE OF SOLUTION

Some results for wheat continuous ventilation simulation from August 15, 1970 at Waddingtom (England) are shown in Figs. 1 and 2. Fig. 1 shows the wheat moisture content profile. Fig. 2 shows the corresponding wheat temperature profile. The initial layer height is 3 m and initial wheat moisture content is 20% w.b. The strategy 1 (i.e. continuous fan – J í l e k , 1994) without any heater has been selected. The time period of ventilation is deli-



1. Wheat moisture content profile for continuous ventilation



2. Wheat temperature profile for continuous ventilation

berately higher than it would be under normal ventilation procedure. The purpose is to show what is going on when ventilation have continued but drying stopped. The influence of less favourite weather conditions in the autumn can be seen. Once drying had stopped, higher weather humidity have caused continuous increase of the average wheat moisture content.

Notation

c_a specific heat capacity of dry air, kJ.kg^{-1}.°C^{-1}
c_g specific heat capacity of dry grain, kJ.kg^{-1}.°C^{-1}
c_v specific heat capacity of water vapour, kJ.kg^{-1}.°C^{-1}
c_w specific heat capacity of water, kJ.kg^{-1}.°C^{-1}
G mass rate of flow, $kg_a.s^{-1}.m^{-2}$
h heat transfer coefficient, kW.m^{-2}.°C^{-1}
H absolute humidity of air, $kg_v.kg_a^{-1}$
k drying constant, h^{-1}
l latent heat of vaporization of pure water, kJ.kg^{-1}
l_g latent heat of vaporization of water in grain, kJ.kg^{-1}
M grain moisture content, decimal d.b.
R parameter defined by equation (17)
S specific surface area of grain, m^2.m^{-3}
t time, h
x depth of bed, m
ε void ratio, $m_{av}^3.m^3$
Θ grain temperature, °C
ρ_g dry matter density, $kg_g.m^{-3}$
ρ_a dry air density, $kg_g.m^{-3}$

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Matematický popis modelu skladování a sušení zrna.

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Ve spolupráci Silsoe Research Institute s Vysokou školou zemědělskou v Praze byl vyvinut model STOREDRY, který popisuje ventilaci zrna studeným nebo přihřátým vzduchem. Model je možné použít pro sušení i skladování různých druhů zrnin. Hlavní částí modelu je řešení čtyř parciálních diferenciálních rovnic, které popisují konvekční přenos tepla a přenos vlhkosti ve ventilované vrstvě zrnitého materiálu. Model obsahuje dvě metody řešení rovnic. První metoda je založena na předpokladu vzniku rovnováhy mezi zrnem a prostředím. Podstatou rovnovážné metody je předpoklad, že proces nízkoteplotního sušení je pomalý proces probíhající dny, týdny a v některých případech i měsíce. Tudíž je možné předpokládat, že zrno dosáhne rovnováhy (pokud se jedná o teplotu a vlhkost) s prostředím (vzduchem) během každého časového kroku integrace diferenciálních rovnic. Druhá metoda obsahuje poněkud náročnější řešení diferenciálních rovnic podle Nellista. Z porovnání obou metod vyplývá, že řešení vede k téměř shodným výsledkům.

Některé výsledky simulace ventilace a sušení pšenice pro období od 15. srpna 1970 v lokalitě Waddington (Anglie) jsou zobrazeny na obr. 1 a 2. Obr. 1 znázorňuje vlhkostní profil zrna, obr. 2 odpovídající teplotní profil zrna. Počáteční výška vrstvy byla zvolena 3 m a počáteční vlhkost pšenice 20 %. Vybrána byla strategie nepřerušovaného chodu ventilátoru bez ohřívače vzduchu. Doba chodu ventilátoru byla záměrně zvolena delší, než by odpovídalo skutečnosti. Účelem bylo totiž ukázat, jaké procesy probíhají, jestliže ventilaci prodloužíme za dobu, kdy sušení skončilo. Z obrázků je zřejmý vliv méně příznivých podzimních podmínek. Jakmile sušení skončilo, vyšší vlhkost vzduchu způsobuje vzestup průměrné vlhkosti pšenice ve vrstvě.

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