DECISION MAKING IN RISK ENVIRONMENT BASED ON FUZZY LINEAR PROGRAMMING

J. Havlíček, P. Kučera

Czech University of Agriculture, Faculty of Economics and Management, Prague, Czech Republic

To deal with imprecision and risk, the concepts and techniques of fuzzy sets are employed. They are successful in managerial decision making, in artificial intelligence and expert systems and many other fields of solving of ill-structured problems. They, unlike confusing stochastic models, provide the best ways to communicate with experts and fuzzy possibilistic programming techniques are considered to be more efficient and meaningful. The risk of the acceptable solution can be expressed by means of (i) probability, (ii) subjective probability, (iii) fuzzy measure. Both cases (i), (ii) use stochastic linear programming. To express risk by means of fuzzy measure, fuzzy linear programming model is used and the risk of considered solution is expressed on a degree scale ⟨0, 1⟩. Basic computational procedures are illustrated using the model of a farm.

fuzzy decision making; fuzzy linear programming; possibilistic programming; stochastic programming; risk of the acceptable solution; measure of the risk; decision support systems

INTRODUCTION

Describing complex real-world systems into precise mathematical models is the main trend in contemporary science and engineering. Real world situations are often not deterministic and thus deterministic mathematical models are not enough to tackle all practical problems. To deal with imprecision and uncertainty, the concepts of probability theory were usually employed. Practical experience proved that the probability mathematical models can be replaced by fuzzy mathematical models which represent better the individual experience. Fuzzy approach can substitute well subjective probability by means of fuzzy measure and enables use the simpler mathematical procedures during computation process.

In the 1960, meanings of the probability theory have been reconsidered and criticised when modelling problems in artificial intelligence and ill-structured problems of management decision making. About the same time as the development of chaos theory, fuzzy set theory was developed in 1965 by L. A. Zadeh and since then, fuzzy set theory and concepts have been used to solve important real world problems including production, manufacturing, transportation, assignment, game, environmental management, water resources management, nutrition management, personnel management, logistics, accounting, banking, finance, marketing management, trade balance and agricultural economics. It should be noted that all concepts and methods are developed for solving practical problems (H w a n g , 1978).

In the last 30 years, the fuzzy set techniques have been applied in many disciplines: operation research, management science, expert systems, control theory, etc. Both techniques and application were systematically discussed. Since 1980s, the possibility theory has become more and more important in the fields of applied management science and impacts on the extension of possibilistic mathematical programming. In operation research and system analysis, fuzzy set theory has been applied to techniques of linear and nonlinear programming, dynamic programming, queuing theory, multiple criteria decision making, group decision making and building effective intelligent decision support systems within the analysis of soft systems.

Fuzzy set theory is a theory of graded concepts and not a theory of chance. Therefore, figures and numerical tables are considered paramount in the study and they (unlike mathematical difficulties) provide the best ways to communicate with non mathematics and non computing people and experts. This is a new important concept in decision support systems.

Concepts in fuzzy linear programming

Symbolically, the general linear programming problem may be stated as: maximise

$$z = \mathbf{cx} \tag{2.1}$$

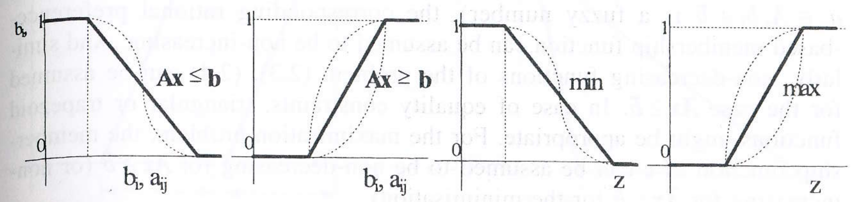
subject to conditions

$$\mathbf{Ax} \le \mathbf{b},\ \mathbf{x} \ge 0 \tag{2.2}$$

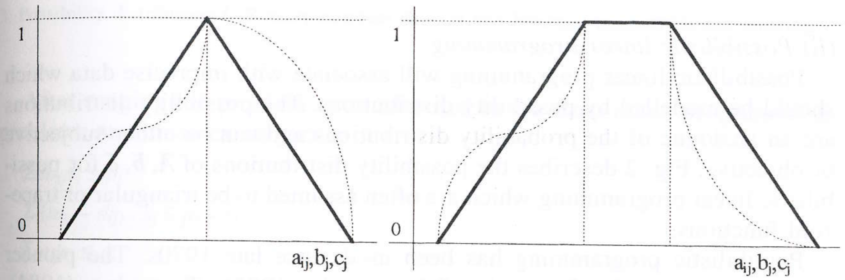
The matrix $\mathbf{A} = (a_{ij})$ is the matrix of technical coefficients, the vector $\mathbf{b} = (b_i)$ is the vector of right-hand sides, the vector $c = (c_{ij})$ represents the costs of decision/slack variables $\mathbf{x} = (x_j)$. All data $\{a_{ij}, b_i, c_j\}$ are considered to be deterministic in the classic LP problem.

These impute data $\{\mathbf{A}, \mathbf{b}, \mathbf{c}\}$ can be fuzzy (imprecise) because of incomplete or non-obtainable information. To formulate these as fuzzy numbers, we can use membership functions or possibility distribution depending on

specificity of the problem. The functional forms of membership functions and possibility distributions are depicted for different types of linear programming problems in Figures 1 and 2, respectively.



1. The membership functions for different types of restricting conditions and objective functions



2. The possibility distribution functions used to describe the fuzzy numbers in LP model

With these fuzzy impute data, equations (2.1), (2.2) are then called fuzzy/possibilistic LP programming. Fuzzy impute data $\mathbf{A} = (a_{ij})$, $\mathbf{b} = (b_i)$, $\mathbf{c} = (c_j)$ we denote by $\tilde{A} = (\tilde{a}_{ij})$, $\tilde{b} = (\tilde{b}_i)$, $\tilde{c} = (\tilde{c}_j)$, respectively. General fuzzy/ /possibilistic linear programming problem may be stated as: maximise

$$z = \tilde{c}\,x \tag{2.3}$$

subject to conditions

$$\tilde{A}\,x \le \tilde{b},\ x \ge 0 \tag{2.4}$$

The grade of a membership function indicates a subjective degree of satisfaction within given tolerances and, on the other hand, the grade of possibility indicates the subjective or objective degree of occurrence of an event. The use of the type of membership function realizes two different concepts in fuzzy mathematical programming:

i) Fuzzy linear programming

Fuzzy programming problems associate fuzzy impute data which should be modelled by subjective preference-based membership functions.

Fig. 1 illustrates different cases of the preference membership functions. When the constraints of the problem (2.4) $\tilde{A}x \leq \tilde{b}$ are (at least one $a_{ij} \in \tilde{A}$, $b_i \in \tilde{b}$ is a fuzzy number), the corresponding rational preference-based membership function can be assumed to be non-increasing. And similarly, non-decreasing functions of the problem (2.3), (2.4) can be assumed for the case $\tilde{A}x \geq \tilde{b}$. In case of equality constraints, triangular or trapezoid functions might be appropriate. For the maximisation problem, the membership function of **c** can be assumed to be non-decreasing for $\tilde{A}x \geq \tilde{b}$ (or non-increasing for $\tilde{A}x \leq \tilde{b}$ for the minimisation).

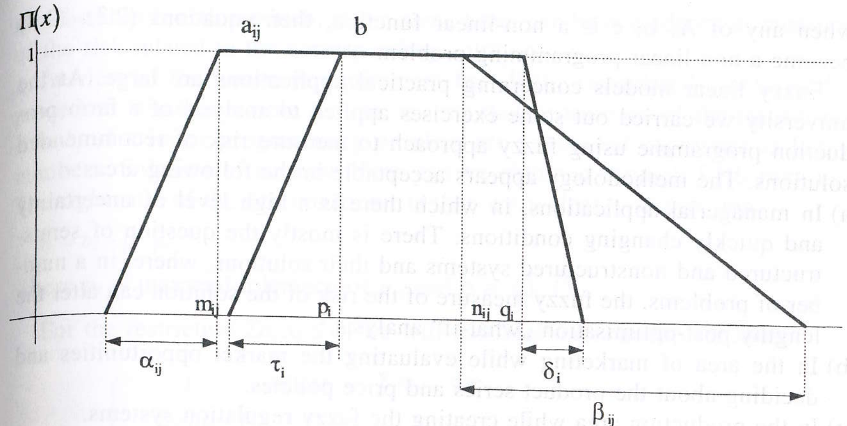
The most recent literature surveys of fuzzy LP are works of Zimmermann (1978, 1983, 1985), Lai, Hwang (1992a, b), Luhamdjula (1989) and Slowinski (1986).

(ii) Possibilistic linear programming

Possibilistic linear programming will associate with imprecise data which should be modelled by possibility distributions. The possibility distributions are an analogue of the probability distributions and can be either subjective or objective. Fig. 2 describes the possibility distributions of $\tilde{A}, \tilde{b}, \tilde{c}$ for possibilistic linear programming which are often assumed to be triangular or trapezoid functions.

Possibilistic programming has been used since late 1970s. The pioneer works were done by Ramik, Rimanek (1985), Tanaka (1984), Dubois (1987), Lai (1992), Hanuscheck, Wolf (1989), etc. Since Zadeh (1978) there has been much research on the possibility theory. Possibilistic decision making models have provided an important aspect in handling practical decision making problems. Unlike stochastic linear programming possibilistic linear programming provide more computational efficiency and is more flexible to use. The possibility measure of an event might be interpreted as the possibility degree of its occurrence under a possibility distribution $\Pi(x)$ analogous to a probability distribution in stochastic linear programming $p(x)$.

In possibilistic linear programming imputes $\{\tilde{A}, \tilde{b}, \tilde{c}\}$ may be imprecise with possibilistic distributions where $\tilde{a}_{ij}, \tilde{b}_i, \tilde{c}_{ij}$ are *L–R* fuzzy numbers. Although there are many different expressions for the *L–R* fuzzy numbers (triangular, trapezoid, linear and nonlinear functions), we can use tetrad expression $\tilde{a}_{ij} = (m, n, \alpha, \beta)$, where m is the left main value and n is the right main value with complete membership, α is the left spread and β is the right spread (see Fig. 3).



3. Possibility distribution *L–R* fuzzy numbers (by Lai and Hwang): trapezoid linear function

After ranging the problem is transformed to the crisp linear programming problem: maximise **cx** subject to

$\Sigma\, m_{ij}x_j \leq p_i$

$\Sigma\,(m_{ij} - d_{ij})\,.\,x_j \leq p_i - \tau_i$

$\Sigma\, n_{ij}x_j \leq q_i$

$\Sigma\,(n_{ij} + \beta_{ij})\,.\,x_j \leq q_i + \delta_i$

$x \geq 0$

Here p_i, q_i, τ_i, δ_i are parameters of ranging process. The optimal solution of the possibilistic linear programming is unique and is obtained by means of standard simplex method (Lai, Hwang, 1992b).

When we are solving the stochastic programming problem, the basic idea is to convert the random nature of the problem into an equivalent deterministic situation. After steps performing this idea we obtain deterministic non-linear model. Obviously, it is not easy to solve it. On the other hand, possibilistic programming provides more efficient techniques to solve the imprecise nature of **A**, **b**, **c** and also preserves the original linear model. Besides, membership functions and possibility distributions provide more flexible and meaningful representation of imprecision and uncertainty.

To involve real applications with real decision makers it is possible to adopt also non-linear functions, such as piece-wise linear, exponential, power, hyperbolic, artificial, etc., as indicated in Figs. 1 and 2. Of course, when any of **A**, **b**, **c** is a non-linear function, then equations (2.3), (2.4) become a non-linear programming problem.

Fuzzy linear models concerning practical applications are large. At the university we carried out some exercises applied to analysis of a farm production programme using fuzzy approach to measure risk of recommended solutions. The methodology appears acceptable in the following areas:

a) In managerial applications, in which there is a high level of uncertainty and quickly changing conditions. There is mostly the question of semistructured and nonstructured systems and their solutions, where, in a number of problems, the fuzzy measure of the risk of the solution can alter the lengthy post-optimisation „what-if" analysis.
b) In the area of marketing while evaluating the market opportunities and deciding about the product series and price policies.
c) In the production area while creating the fuzzy regulation systems.

Fuzzy linear programming

There are presented the following main different approaches to solving fuzzy linear programming in the literature:

(i) *Nonsymmetric model equivalent to the parametric programming (Verdegay, 1982).*
(ii) *Werner nonsymmetric model with fuzzy objective (Werner, 1987).*
(iii) *Chanas nonsymmetric model equivalent to the goal programming (Chanas, 1984).*
(iv) *Lai an Hwang interactive LP (Lai, Hwang, 1992c).*
(v) *Zimmerman symmetric model equivalent to the goal programming (Zimmerman, 1976).*

Zimmermans' approach is representative and illustrates well fuzzy concepts in linear programming. It should be noted that this approach is considered as the first practical method to solve a linear programming problem with fuzzy constraints and objective.

We shall show now how to use Verdegay nonsymmetric model to measure the risk of decision making when using linear programming model with fuzzy right sides in constraints. The risk is usually measured in the Bayesian sense.

Let us assume we have to select a variant of solution so as to be in the position to modify the risk of the decision. This demand determines a fuzzy set M = {Solution with risk}, which is defined upon a set of feasible solutions of the problem.

If there is at least one parameter of a linear programming model a fuzzy number, the objective function is a fuzzy number, too. This means, that a given objective function value can be reached with a certain degree of

membership function which, at the same time, can be considered as measure of the risk related to the corresponding solution.

In view of the linearity requirement for all the considered functions the procedure is rather simple. Let us assume that restrictions of the right hand side b_j of a linear programming problem are vague and expressed as fuzzy numbers. Solution of such problem can be obtained through RHS ranging. The right side of the i-th constraint is than equivalent to expression

$$\bar{b}_i = b_i + \rho \,.\, p_i$$

where p_i is maximal tolerance of b_i, and $\rho \in \langle 0, 1\rangle$.

For the restriction $\Sigma a_{ij}x_j \le b_i$ we will consider membership function

$$\mu_i(x) = \begin{cases} 1 & \text{where } \Sigma a_{ij}x_j < b_i \\ 1 - \dfrac{\Sigma a_{ij}x_j - b_i}{p_i} & \text{where } b_i \le \Sigma a_{ij}x_j \le b_i + p_i \\ 0 & \text{where } \Sigma a_{ij}x_j > b_i + p_i \end{cases}$$

Here $\mu(x) = 1$, if the restriction is satisfied, $\mu(x) \in \langle 0, 1\rangle$, if maximum tolerance p_i goes gradually down up to zero value, $\mu(x) = 0$ if the restriction is certainly not satisfied. If the tolerance p_i is defined, the linear programming problem can be formulated as follows:

$$\max \left\{c^{\rho}\, x^{\rho} \mid \Sigma a_{ij}x_j \le b_i + (1-\lambda)\, p_i,\ x_j \ge 0,\ \lambda \in \langle 0, 1\rangle\right\} \qquad (2.5)$$

The problem (2.5) is a problem of RHS ranging with parameter $\rho = 1 - \lambda$. The value ρ is the degree of membership function M = {Solution with risk} and is thus the measure of the risk of the chosen solution. If $\rho = 1$, than $\lambda = 0$ and maximal tolerance p_i reaches its maximum value, i.e. the solution is charged with maximum risk. If than $\rho = 0$, than $\lambda = 1$, the restriction participates in the solution with a value $b_i + 0p_i = b_i$, and the solution is without risk.

In the case of the costs c_j or technical coefficients a_{ij} defined as fuzzy numbers, analogical costs or coefficients ranging has to be applied.

Solution procedure

Step 1: We formulate a deterministic linear programming model.

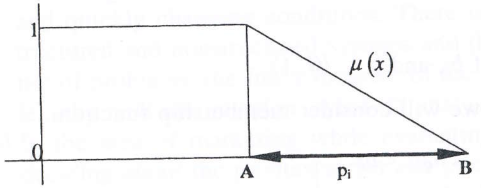
Step 2: We identify restrictions, the RHS of which can be defined as fuzzy numbers.

Step 3: Let the i-th restriction b_i is a fuzzy number. We specify the maximum tolerance p_i. If we cannot determine it directly from the problem formulation, we use an expert estimation. In this case, experts fix up two points:

a) Point A which is the most to the right but belongs still to the fuzzy set with the degree 1.

b) Point B, which is the most to the right and still belongs to the fuzzy set with the nonzero.

The distance between the two points equals to the maximal tolerance p_i as shown in Fig. 4.



4. Maximal tolerance p_i in relation with risk measured in interval ⟨0, 1⟩

Step 4: We carry out the computation using standard simplex algorithm for values imputed in the first step of the procedure. The final simplex table is shown below:

Z	$x_1, \ldots, x_n$	$s_1, \ldots, s_m$	RHS
	A	$\mathbf{B}^{-1}$	β
$z_j - c_j$	d_j	d_k	z_{max}

Where: Z – set of indices of basic variables
x_j – decision variables
s_k – slack variables
A – matrix of technical coefficients
$\mathbf{B}^{-1}$ – transformation matrix
d_j, d_k – shadow prices, dual values
$\beta_i \in \beta$ – coordinates of the basic solution

Step 5: We solve the RHS ranging problem for $b_i = b_i + \rho p_i$ i.e. determine

$$b_i^{\rho} = \boldsymbol{B}^{-1}(b_i + \rho p_i) = \beta + \rho \boldsymbol{B}^{-1} p_i$$

For $\rho \in \langle 0, 1 \rangle$ we obtain the values of the solution vector with the risk of degree $\mu(x) = \rho$.

Step 6: We carry out the analysis and synthesis of the obtained results (Havlíček, 1992).

Illustration by example

We shall follow steps in Solution procedure above:

Step 1. The model of a farm below illustrates the above procedure, its original formulation is shown in Tab. I. The following activities are included in the model: 5 types of crop production, 4 categories of cattle raising, sales of grains and purchases of feed blendings. The constraints 1–5 describe the use of arable land and the crop rotation, the constraints 6–10 formalise the balance of nutrients, the constraints 11–15 give the capacity limitations and cattle reproduction. The four remaining constraints set up limits on purchases of feed blendings, labour cost and material cost and obligation for sales. The objective is profit maximisation. Activities and constraints in crop production are converted per ha, in animal production per breeding unit and piece of animal, other indicators in kind are expressed in metric tons, those in money in thousands of Kc (Czech crowns).

Steps 2–4. It can be supposed that the farm has more sources at disposal. The stock of feed concentrates (see the RHS of the constraint No. 8) may reach up to 55, i.e. the maximum tolerance $p_8 = 40$.

There are 100 Th. of Kc available for increase of labour cost or material expenditure. The final solution of the model is shown in Tab. II. It follows from the table that an increase of labour cost may be proper, since its capacity is fully used in the final solution, while the capacity of material expenditure is not. The second fuzzy number in the RHS vector will therefore be the RHS of the constraint no. 17, maximum tolerance $p_{17} = 100$. In the worst situation, the farm will be in the position to spend 1550 Th. of Kc on labour cost. The vector-column of maximum tolerances p_i is shown in Tab. I on its right column.

In accordance with the previous text, we shall denote the rate of risk of the acceptable solution by ρ. If there are at farm disposal the stock of 15 tons of feed concentrates and the 1 450 Th. of Kc for labour cost, these values are corresponding to the rate of risk $\rho = 1 - \lambda = 0$. If the farm counted with the maximum level of sources, i.e. 55 tons of feed-concentrates and 1 550 Th. of Kc on labour cost, it would run the maximum possible risk, thus $\rho = 1 - \lambda = 1$. The risk of rate $\rho = 0.5$ means, for instance, that the farm relies, in fact, on the stock of 35 t of feed-concentrates and on financial means of 1 500 Th. Kc on labour cost.

Step 5. To find out, how the optimum solution would react upon the growing rate of risk , the RHS ranging of the problem along parameter $\rho \in \langle 0; 1 \rangle$ was carried out, see Tabs. II 2 and III. In agreement with the 5-th step of the algorithm for various grades of the parameter ρ the solutions were specified. The optimal solutions for different values of risk ρ are presented

I. Initinal simplex table

Constr.		X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11		Bound	Max. tol.
	B	A. wheat	Oats	S. beet	Maize	Per. f. c	Cows	Calves	Heifers	Fatt. cattle	Cer. sales	P. blend.		b	p
1	A1 Arable	1	1	1	1	1							=	400	0
2	S2 Oats a.		1										>=	30	0
3	S3 S. beet a.			1									<=	80	0
4	S4 Maize a.				1								>=	50	0
5	S5 Per. f. c.a					1							<=	100	0
6	S6 Nitrogen	0.22	0.16	0.24	0.24	0.52	–0.36	–0.12	–0.45	–0.37	–0.08	0.015	>=	20	0
7	S7 Starch	2.02	1.29	2.53	2.7	2.1	–2.17	–0.6	–2.74	–2.05	–0.71	0.06	>=	50	0
* 8	A8 Conc.	–4.3	–2.25				0.7	0.1	0.5	1	1		=	15	40
9	S9 Blend.						0.6	0.1				–1	<=	0	0
10	S10 Rough.			–45	–36	–41	1.6		1.7	1.5			<=	0	0
11	S11 C. cows						1						<=	80	0
12	S12 R. cows						–0.25		0.7				>=	0	0
13	A13 B. calves						–0.96	1.1					=	0	0
14	A14 B. br. cattle							1	–1.25	–1.05			=	0	0
15	S15 C. y. cattle							0.6	2	1.6			<=	150	0
16	S16 Limit. bl.											1	<=	120	0
* 17	S17 L. cost	1.2	1.4	5	2	1.5	7.8	1.1	3.1	1.8			<=	1 450	100
18	S18 M. cost	7.8	7.6	24.7	9.7	5.2	20	6.4	10.2	6.7		3.2	<=	12 000	0
19	S19 Sales			31.5			44.5	6.4	16.4	15.5	3		>=	8 000	0
	OF Profit	–9	–9	1.8	–11.7	–6.7	16.7	–1.1	3.1	7	3	–3.2		,	

X1, X2,..., X11 – decision variables
S2, S3, S4, S5, S6, S7, S9, S10, S11, S12, S15, S16, S17, S18, S19 – slack variables
A1, A8, A13, A14 – artificial variables
Objective function (OF) – profit maximization
Arable – arable land; Oats a. – area of oats; S. beet a. – area of sugar beets; Maize – area of maize; Per. f. c. – perennial fodder crops; Nitrogen – balance of nitrogen matters; Starch – balance of starch units; Conc. – feed concentrates; Blend. – balance of feed-blendings; Rough. – balance of roughage; C. cows – capacity for milking cows; R. cows – reproduction of milking cows; B. calves – balances of calves; B. br. cattle – balance of breeding cattle; C. y. cattle – capacity for young cattle; Limit. bl. – limitation for feed-blendings; L. cost – labour cost; M. cost – material expenditure; Sales – sales
A. wheat – autumn wheat; Oats – oats; S. beet – sugar beet; Maize – maize silage; Per. f. c.a – perennial fodder crops; Cows – milking cows; Calves – calves; Heifers – heifers; Fatt. cattle – cattle for fattening; Cer. sales – sales of cereals; P. blend – purchases of feed-blendings

II. Final simplex table

B	A1 Arable l.	S2 Oats a.	S3 S. beet a.	S4 Maize a.	S6 Nitrogen
X1 A. wheat	0.565556	0.501773	−0.84603	−0.18852	0
X2 Oats	0	−1	0	0	0
X3 S. beet	0	0	1	0	0
X4 Maize	0	0	0	−1	0
X5 Per. f. c.	0.434444	0.498227	−0.15397	1.188519	0
X6 Cows	−0.12553	0.004768	−0.3542	0.041843	0
X7 Calves	−0.10955	0.004161	−0.30912	0.036517	0
X8 Heifers	−0.04483	0.001703	−0.1265	0.014944	0
X9 Fatt. cattle	−0.05096	0.001936	−0.14381	0.016988	0
X10 Cer. sales	2.604093	−0.09892	−3.15202	−0.86803	0
X11 P. blend.	−15.9582	−14.2455	−33.3807	−24.9028	−66.6667
S5 Per. f. c.a	−0.43444	−0.49823	0.153971	−1.18852	0
S7 Starch	−0.1862	−0.03612	2.328275	−1.65127	−4
S9 Blend.	−15.8719	−14.2488	−33.1373	−24.9316	−66.6667
S10 Rough.	18.16573	20.41386	39.68468	12.61142	0
S11 C. cows	0.125528	−0.00477	0.354202	−0.04184	0
S15 C. y. cattle	0.236936	−0.009	0.668563	−0.07898	0
S16 Limit. bl.	15.95818	14.24552	33.38071	24.90283	66.66667
S18 M. cost	48.40616	46.52871	100.8342	83.34239	213.3333
OF Profit	48.40616	46.52871	100.8342	83.34239	213.3333

in Tab. III. For ρ = 0.05 and ρ = 0.8 the change of basis is needed, thus there are three intervals of the stability of solutions, namely (i) for value of risk ρ ∈ ⟨0; 0.05⟩, (ii) for ρ ∈ ⟨0.05; 0.08⟩, (iii) ρ ∈ ⟨0.8; 1⟩. The values of activities change more or less along risk parameter ρ and the rate of their change enables a better judgement of the system behaviour around the borderline of stability intervals for the value ρ ∈ ⟨0; 1⟩. Based on this information particular results obtained can be summarised.

Step 6. In the optimum solution of the original (not fuzzy) model, while the rate of risk is zero, the feed-blendings are very unbalanced (i.e. much more blendings are fed than needed). In the first interval of stability, at very low values of the rate of risk ρ, up to 0.05 and with its growth, the difference

II. Final simplex table – continue

A8 Conc.	S12 R. cows	A13 B. calves	A14 B. br. cattle	S17 L. cost	S19 Sales	β
−0.20625	−0.51367	−0.16069	0.172116	−0.37704	−0.06875	153.1237
0	0	0	0	0	0	30
0	0	0	0	0	0	80
0	0	0	0	0	0	50
0.206253	0.51367	0.160688	−0.17212	0.377037	0.068751	86.87628
−0.00584	0.114479	−0.24596	0.166629	0.083685	−0.00195	56.04269
−0.0051	0.099909	0.694437	0.145421	0.073034	−0.0017	48.90999
−0.00209	−1.38769	−0.08784	0.05951	0.029888	−0.0007	20.01525
−0.00237	1.747159	0.765942	−0.88473	0.033976	−0.00079	22.75326
0.121121	−3.35222	−1.31025	1.463891	−1.73606	−0.29296	664.0502
−3.781	−23.1391	5.708703	−3.62583	−12.4723	−3.03811	115.418
−0.20625	−0.51367	−0.16069	0.172116	−0.37704	−0.06875	13.12372
−0.27006	0.945042	0.073229	−0.06888	0.137468	0.039981	100.8549
−3.77699	−23.2178	5.786834	−3.74035	−12.5298	−3.03677	76.90142
8.472824	20.61562	5.982161	−6.09744	15.22285	2.824275	8804.103
0.005838	−0.11448	0.245958	−0.16663	−0.08369	0.001946	23.95731
0.01102	−0.08003	−1.46649	1.209295	−0.15796	0.003673	44.21829
3.781001	23.13912	−5.7087	3.625831	12.47232	3.038111	4.581962
12.82199	74.90016	−21.6111	12.21257	38.21812	9.962886	5505.06
12.82199	74.90016	−21.6111	12.21257	37.21812	8.962886	55.06049

sharply declines, at the same time the area of perennial fodder-crops is increasing on expense of the winter wheat and the bulk of all activities in animal production is expanding, on the other hand, the sales of cereal are going down and the material expenditure is sinking quickly as well, an equally dynamic growth shows the profit. This means that an increase of available labour, which is accompanied with a growing rate of risk, is the most advantageous for the use in animal production. This makes it also possible, in combination with second fuzzy number – the stock of feed – concentrates – to cut down the purchases of expensive feed-blendings. It is worth to notice that the values of coefficients in the final simplex table are rather high, which denotes a high level of sensibility of the optimum solution towards the impute data.

III. Solution of model depending on membership function

ρ	0	0.05	0.1	0.2	0.3
X1 A. wheat	153.1237	150.826	149.9661	148.5494	147.1328
X2 Oats	30	30	30	30	30
X3 S. beet	80	80	80	80	80
X4 Maize	50	50	50	50	50
X5 Per. f. c.	86.87628	89.17397	90.03394	91.45059	92.86725
X6 Cows	56.04269	56.44944	56.89689	57.80036	58.70384
X7 Calves	48.90999	49.26497	49.65546	50.44395	51.23244
X8 Heifers	20.01525	20.16051	20.32032	20.64299	20.96566
X9 Fatt. cattle	22.75326	22.9184	23.10007	23.46688	23.83369
X10 Cer. sales	664.0502	655.6122	653.3005	649.9694	646.6384
X11 P. blend.	115.418	45.49442	39.10368	39.72461	40.34555
S2 Oats a.	0	0	0	0	0
S3 S. beet a.	0	0	0	0	0
S4 Maize a.	0	0	0	0	0
S5 Per. f. c.a	13.12372	10.82603	9.966058	8.549405	7.132753
S6 Nitrogen	0	0	0	0	0
S7 Starch	100.8549	101.0021	100.3133	98.75919	97.20509
S9 Blend.	76.90142	6.698262	0	0	0
S10 Rough.	8 804.103	8 897.163	8 931.162	8 986.7	9 042.239
S11 C. cows	23.95731	23.55056	23.10311	22.19964	21.29616
S12 R. cows	0	0	0	0	0
S15 C. y. cattle	44.21829	43.45055	42.60598	40.90065	39.19532
S16 Limit. bl.	4.581962	74.50558	80.89632	80.27539	79.65445
S17 L. cost	0	0	0	0	0
S18 M. cost	5 505.06	5 721.795	5 730.186	5 703.018	5 675.849
S19 Sales	0	0	20.91196	67.14732	113.3827
OF Profit	55.06049	266.7951	291.0981	300.165	309.2319

Solutions with risk: for ρ = 0 without risk, for ρ > 0 with risk

The rate of risk ρ = 0.05 represents the very moment, when the feed-blendings become well-balanced. If the risk for the chosen solution continues to increase, the feed-blendings will only be used in the minimum necessary size.

III. Solution of model depending on membership function – continue

0.4	0.5	0.6	0.7	0.8	0.85	0.9	1
145.7161	144.2994	142.8828	141.4661	140.0495	140	140	140
30	30	30	30	30	30	30	30
80	80	80	80	80	80	80	80
50	50	50	50	50	50	50	50
94.2839	95.70055	97.1172	98.53386	99.95051	100	100	100
59.60732	60.5108	61.41427	62.31775	63.22123	63.0258	62.78175	62.29365
52.02093	52.80942	53.59791	54.3864	55.17489	55.00433	54.79134	54.36536
21.28833	21.611	21.93367	22.25634	22.57901	22.50921	22.42205	22.24773
24.2005	24.56731	24.93412	25.30093	25.66774	25.5884	25.48931	25.29114
643.3074	639.9763	636.6453	633.3142	629.9832	632.0385	634.3733	639.0429
40.96648	41.58742	42.20836	42.82929	43.45023	43.31591	43.14818	42.81272
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
5.716101	4.299448	2.882796	1.466143	0.049491	0	0	0
0	0	0	0	0	0	0	0
95.651	94.0969	92.5428	90.98871	89.43461	88.85159	88.28314	87.14624
0	0	0	0	0	0	0	0
9 097.777	9 153.316	9 208.854	9 264.393	9 319.931	9 322.51	9 323.198	9 324.572
20.39268	19.4892	18.58573	17.68225	16.77877	16.9742	17.21825	17.70635
0	0	0	0	0	0	0	0
37.48999	35.78465	34.07932	32.37399	30.66866	31.03754	31.49819	32.41949
79.03352	78.41258	77.79164	77.17071	76.54977	76.68409	76.85182	77.18728
0	0	0	0	0	7.05636	14.64278	29.81563
5 648.681	5 621.512	5 594.344	5 567.175	5 540.007	5 546.809	5 555.143	5 571.81
159.618	205.8534	252.0888	298.3241	344.5595	338.5624	330.3782	314.0098
318.2988	327.3657	336.4326	345.4995	354.5664	357.428	360.164	365.636

At the same time the situation arises, in which the increasing labour resources cannot be used efficiently anymore and remain partly unused. In the third interval of stability when ρ > 0.8 the change of optimum programme at the growing rate of risk is characterised by two substantial processes: (i) perennial crops uses all area of disposal area (100 ha) and (ii) the labour cost are decreasing. This would logically originate, and it is also confirmed by the computation results, the growth of the bulk of sold cereals, extension of the winter wheat area on expense of area of perennial fodder-crops and the decline of animal production. These tendencies are contradictory in relation to the first interval of stability and their changes are considerably less noticeable. Also the expenditure is decreasing and the profit increasing, in this interval, much slower, the lower coefficients in Tab. II confirm the less important influence of eventual modifications of production imputes on the resulting production.

The farm can be advised to run a sound risk and choose the rate of risk from the second interval of stability, where the profit has already been increased by the important growth from the first interval, eventual modifications of the original constraints are of a rather small importance. This choice becomes even more advantageous, if the farm can employ a part of the unused means destined to purchases of a higher stock of feed-concentrates.

References

CHANAS, S.: A fuzzy linear programming problem with equality constraints. Control and Cybernetics, *13*, 1984: 195–201.

DUBOIS, D.: Linear programming with fuzzy data. In: Analysis of Fuzzy Information. CRC Press 1987: 241–263.

HANUSCHECK, R.: Linear programming with fuzzy objectives. Fuzzy Sets and Systems, *29*, 1989: 31–48.

HAVLÍČEK, J.: An interactive goal programming. In: Proc. Conf. Agrar Perspectives I, Univ. of Agriculture, Prague, 1992: 339–346.

HWANG, CH. L. – LAI, Y. J.: Fuzzy Mathematical Programming. Springer 1992a.

LAI, Y. J.: A new approach to some possibilistic programming problem. Fuzzy Sets and Systems, *50*, 1992: 235–241.

LAI, Y. J. – HWANG, CH. L.: Interactive fuzzy linear programming. Fuzzy Sets and Systems, *45*, 1992a: 67–86.

LAI, Y. J. – HWANG, CH. L.: Possibilistic linear programming for managing interest rate risk. Fuzzy Sets and Systems, *49*, 1992b: 162–171.

LAI, Y. J. – HWANG, CH. L.: IFLP–II: a decision support system. Fuzzy Sets and Systems, *50*, 1992c: 227–234.

LUHANDJULA, M. K.: Fuzzy optimization and appraisal. Fuzzy Sets and Systems, *30*, 1989: 257–282.

RAMIK, J. – RIMANEK, J.: Inequality relation between fuzzy numbers and its use in fuzzy optimization. Fuzzy Sets and Systems, *16*, 1985: 123–138.

SLOWINSKI, R. A.: A multicriteria fuzzy linear programming method for water supply system development planning. Fuzzy Sets and Systems, *19*, 1986: 217–237.

TANAKA, H. H.: A formulation of fuzzy linear programming based on comparison of fuzzy numbers. Control and Cybernetics, *13*, 1984: 186–194.

VERDEGAY, J. L.: Fuzzy mathematical programming. In: Approximate Reasoning in Decision Analysis, North-Holland, 1982: 231–236.

WERNERS, B.: Interactive multiple objective programming subject to flexible constraints. Eur. J.O.R., *31*, 1987: 342–349.

ZADEH, L. A.: Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets and Systems, *1*, 1978: 3–28.

ZIMMERMANN, H. J.: Description and optimization of fuzzy system. Int. J. Gen. System, *2*, 1976: 209–216.

ZIMMERMANN, H. J.: Fuzzy programming and linear programming with several objective functions. Fuzzy Sets and Systems, *1*, 1978: 45–55.

ZIMMERMANN, H. J.: Using fuzzy sets in operational research. Eur. J.O.R., *13*, 1983: 201–216.

ZIMMERMANN, H. J.: Application of fuzzy set theory to mathematical programming. Inf. Sci., *36*, 1985: 29–58.

Received for publication on June 16, 1997

HAVLÍČEK, J. – KUČERA, P. (Česká zemědělská univerzita, Provozně ekonomická fakulta, Praha, Česká republika):

Rozhodování v rizikovém prostředí založené na fuzzy lineárním programování.

Scientia Agric. Bohem., *28*, 1997 (3): 227–244.

Aplikace matematických modelů lineárního programování (LP) jsou v případě deterministických vstupních informací standardním nástrojem podpory rozhodování. V praktických aplikacích se často setkáváme s případy, kdy některé vstupní informace nejsou přesné a řešení je možno přijmout jen s určitým rizikem. Míru rizika, zpravidla uvažovanou a měřenou v Bayesově smyslu, lze do modelu zahrnout pomocí pravděpodobnostní míry nebo fuzzy míry. Fuzzy míra se v matematických modelech používá z těchto důvodů: (i) dobře vyjadřuje osobní zkušenost a cit pracovníka, (ii) lze ji snadno nahradit subjektivní pravděpodobnost, (iii) algoritmy pro řešení modelů s fuzzy prvky jsou jednodušší než v modelech stochastických. Neurčitost/nepřesnost dat se v modelu LP vyjadřuje pomocí fuzzy čísel. Obecně lze funkci rozdělení fuzzy čísla vyjádřit dvěma způsoby: a) funkcí příslušnosti (obr. 1), nebo b) rozdělením možností (possibility distribution – obr. 2). Pro měření rizika je výhodnější použít funkci rozdělení možností (tzv. „posibilistické" LP používá funkci rozdělení zobrazenou na obr. 3; z obrázku je zřejmá formulace omezujících podmínek posibilistického modelu LP).

Cílem práce bylo navrhnout algoritmus pro měření rizika přijaté varianty řešení modelu LP s neurčitými/nepřesnými daty. Jako základní typ modelu fuzzy LP je uvažován Zimmermanův symetrický model, který je blízký modelu cílového programování a dobře vyhovuje základním požadavkům algoritmu: je interaktivní, citlivý

na změnu vstupních informací, algoritmicky jednoduchý. Při řešení modelu fuzzy se využívá známých postupů pro parametrizaci v LP (RHS ranging).

Základní kroky algoritmu:

1. Formulace deterministického modelu LP.
2. Určení vstupních dat, která jsou neurčitá a budou zadána jako fuzzy čís (v uvedeném algoritmu se uvažuje o hodnotách pravých stran b_i).
3. Pro každé fuzzy číslo se určí tzv. maximální tolerance; pro fuzzy číslo b_i je tolerance p_i (obr. 4).
4. Pro zvolené hodnoty tolerance p_i se počítají varianty řešení modelu LP algoritme RHS ranging. Počet hodnot p_i se stanoví tak, aby se varianty řešení od seb dostatečně lišily; v uvedeném příkladě bylo stanoveno 13 hodnot p_i.
5. Pomocí parametru $\rho_i \in \langle 0, 1\rangle$ se pro každou variantu určuje riziko, měřené n intervalu $\langle 0, 1\rangle$. Hodnota "0" představuje absolutní riziko, hodnota „1" jistotu.
6. Provede se analýza výsledků (viz např. Havlíček, 1992).

Popsaný postup je ilustrován na příkladu zemědělské farmy (5 typů plodin, 4 ka tegorie zvířat, nákup a prodej). Matematický model je popsán výchozí a výsledno simplexovou tabulkou (tab. I a II). Neurčitá data představují v tabulce omezení b_8 = 15 (koncentrovaná krmiva – maximální tolerance p_8 = 40, tj. hodnota b_8 se můž pohybovat v rozmezí 15–55) a omezení b_{17} = 1 450 (pracovní náklady v 10^3 Kč - maximální tolerance p_{17} = 100, tj. b_{17} se může pohybovat v rozmezí 1 450–1 550) V tab. III jsou spočítány varianty řešení modelu LP pro hladiny rizika $\rho_1 = 0$; ρ_2 = 0,05; ... ; ρ_{13} = 1.

fuzzy rozhodování; fuzzy lineární programování; posibilistické programování; stochastické programování; riziko přijatého řešení; měření rizika; systémy podpory rozhodování

Contact Address:

Doc. RNDr. Jaroslav Havlíček, CSc., Česká zemědělská univerzita, Provozně ekonomická fakulta, 165 21 Praha 6-Suchdol, Kamýcká 129, Česká republika, tel.: 02/24 38 23 52, fax: 02/39 37 08