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QUALITATIVE LINEAR PROGRAMMING BASED ON THE POST ANALYSIS APPROACH

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The qualitative reasoning in strategic planning is applied to linear programming problem (LP). For short-term planning the LP sales mix problem is used and for long-term planning the special methods for strategic planning are developed. For the support of qualitative judgements the graphic schemes, such as the BCG and GE matrices and trend graphs are organised with the guide of decision support system. Qualitative reasoning uses the post-model analysis approach. Information in the final simplex tableau analysed and used by decision maker, experts or artificial expert system utilise the framework of the procedure. Algorithm is illustrated by an example.

Qualitative reasoning; LP sales mix problem; qualitative judgement; decision support systems; strategic planning; short-term and long-term profits; BCG and EG matrices; trend graphs; post-optimisation analysis

INTRODUCTION

Many practical strategies of planning in agriculture are divided into two groups depending upon the relative length of the planning horizon – short-term planning and long-term planning. For farmer's short-term planning, optimisation models like sales mix models are widely used. If we consider the open production system to be a soft system the long-term planning and various qualitative analysis techniques of strategic planning have been developed (Havlíček, 1996). Usually, these two approaches have been studied separately. Historically, the most common approach to associate two levels of planning is done by adding the constraints which reflect the important considerations of the long-term planning in the short-term optimisation models. This approach, however, is not satisfactory for the following reasons: (i) It is not easy to quantify the strategic considerations to fit the optimisation model due to their vague and uncertain character, and (ii) When conflicts between short-term profit and long-term profit occur, the trade-offs between the quantified short-term profits and the non-quantified long-term profits are unavoidable. To overcome these difficulties, the post analysis of linear programming model (LP) can be used. The methodology was created by Lee (1985) and developed by Havlíček (1973, 1991).

SOLUTION PROCESS

Model

The method presents the opportunity costs of qualitative objectives in terms of the quantitative objective values and supports the evaluation of postoptimisation variants of the solution in terms of qualitative objectives. The two steps are used:

1. For short-term planning, the LP model is used in the form of mix problem.
2. For long-term planning, various qualitative methods for strategic planning are used.

The mix problem is a very popular LP model whose objective function is the maximisation of profits or minimisation of costs subject to capacity constraints and budgetary limits, demands and so on. To respect qualitative requirements, classical LP model is extended as follows:

$$\mathbf{Ax} \ \{\leq, =, \geq\} \ \mathbf{b} \tag{1}$$

$$q_g \ (\mathbf{x} \mid \mathbf{i}) \cup q_c \ (\mathbf{x} \mid \mathbf{i}) \tag{2}$$

$$\mathbf{x} \geq \mathbf{0} \tag{3}$$

$$z = \{\mathbf{cx}, q_g; q_c\} \rightarrow \max \tag{4}$$

In formulas (1) and (4) $\mathbf{x}$ is sales mix vector and $\mathbf{c}$ profit contribution vector, $\mathbf{A} = (a_{ij})$ technological coefficient matrix, $\mathbf{b} = (b_i)$ production capacity vector. The judgement functions (2) are composed of judgement function q_g for the qualitative goal of long-term profit maximisation and judgement function q_c for the qualitative constraints where $i = (i_1, i_2, \ldots, i_n)$ represents the additional information not included in the quantitative parts of the model (1), (4). The first part of the judgement functions (2) concerns objective function, the second one concerns restricting conditions. The judgement functions utilise the additional information by formulating in (i) graphical display form, (ii) by knowledge of experts or an expert system, (iii) by ad hoc inquiry to the data base. The qualitative goals q_g are treated as qualitative constraints and can be computed in terms of $\mathbf{cx}$, while supporting the qualitative judgements. Qualitative constraints q_c have the similar role for the restricting conditions. Thus quantitative variables $\mathbf{x}$ are conditioned on qualitative variables i depending on qualitative human decision.

Framework for processing the judgement functions

The available methods for strategic planning in the literature can be classified in the form presented by Wind (1983) into two groups: Product based models and finance-oriented models. Product based models include standardised models and customised models. Standardised models fit very well with the need of judgements for (2), (4). Standardised models use five standardised methods for strategic planning as shown below:

Uni-dimension models	Growth/share matrix (BCG matrix)
	Growth/gain matrix
Composite dimension models	Business assessment matrix (GE matrix)
	Business profile matrix (Hofer model)
	Directional policy matrix (Shell matrix).

Among the five methods for strategic planning in the standardised models, the BCG matrix represents the single most popular method but it is not complete due to its uni-dimension. To compensate the weakness of BCG's method, the business assessment matrix (GE) and the growth/gain matrix can be used additionally. Since the growth/gain matrix is concerned with the location of products at markets, the graph which displays the trends of the products' sales and market size simultaneously may replace the role of the growth/gain matrix and thus the graph describes additional information about dynamic trend. Business profile matrix and directional policy matrix could be dropped out, because information about market potential and company capability is already utilised in GE matrix. Thus, it is sufficient number to use three methods of strategic planning to evaluate effectively the judgement functions with a minimum degree of duplication: (1) Growth/share matrix, (2) Business assessment matrix, (3) Trend graph.

Algorithms

The algorithms of the solution is depicted in Fig. 1 and can be described as follows:

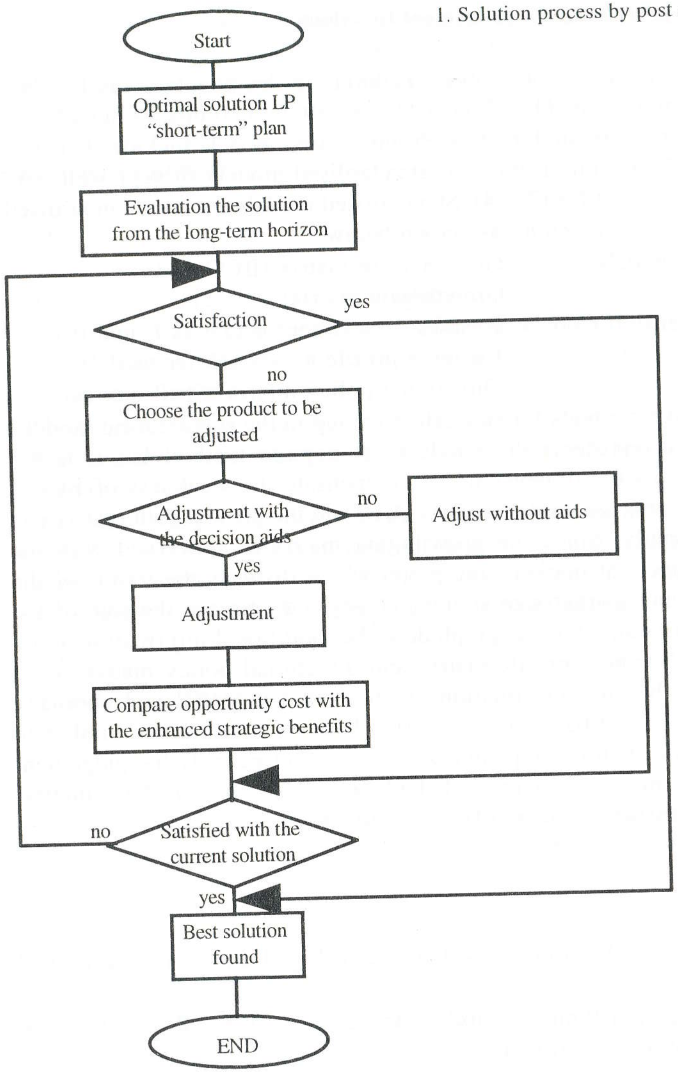
Step 1: We solve the LP model $\max\{z \mid \mathbf{Ax} \ \{\leq, =, \geq\} \ \mathbf{b}, \mathbf{x} \geq \mathbf{0}\}$ and look for quantitatively optimal solution $\mathbf{x}$.

Step 2: The optimal solution $\mathbf{x}$ needs to be evaluated from the point of view of the perspective of its long-term profit. Since the qualitative goal of long-term profits cannot be represented in numeric form, supporting procedures are involved into the solution process:

(i) graphical displays,

(ii) expertise with the guide of experts or expert systems.

1. Solution process by post analysis approach



Step 3: If the current solution fully meets the needs of the user, than the best solution has been found and algorithms finished. In the other case we identify the product that needs adjustment to effectively enhance the strategic benefits.
Step 4: We have to decide about the level of the adjustment using experts and final information of the final simplex tableau. In this process we use

"what-if" analysis using matrix of transformed coefficients, dual prices and shadow prices. The upward and downward adjustment is used to move from the dominated solution to the non-dominated solution on both quantitative objectives and qualitative objectives depending on whether the adjusted solution is feasible to the original quantitative model obtained in the step 1 of the algorithms or not. The outcome of upward/downward adjustment is the opportunity cost. When the adjusted solution is unfeasible to the original quantitative model, decision maker should decide whether to expand the capacity of production facilities to increase the feasibility of the current solution - in this case the required additional investments should be included probably in calculating the opportunity costs. The procedure ends when the opportunity cost of adjustment is determined.
Step 5: The decision maker computes the adjusted solution and the opportunity cost of adjustment.
Step 6: At this point, the decision maker has to decide whether the opportunity cost is acceptable to effectively enhance the strategic benefits.
Step 7: If it is necessary to modify the adjustment, the adjustment procedure may be similarly repeated.

DECISION SUPPORT SYSTEM

The procedure of decision making is supported by decision support system (Havlíček, 1994). Its architecture has the following subsystems:

Quantitative model subsystem. It generates quantitative models for short-term profit maximisation and serves to storage, retrieval and maintenance of the models.

Qualitative model subsystem. It consists of both the knowledge-based expert systems and graphs.

The graphs base contains a set of graphs useful for strategic evaluation:
(1) Graphs describing decision maker's products (BCG matrices, dynamic BCG matrices, GE matrices).
(2) Graphs describing the competitor's products (BCG matrices, dynamic BCG matrices).
(3) Trend graphs (for decision maker's sales versus market size, competitor's sales versus market size, decision maker's sales versus competitor's sales).

Expert system includes:
(1) The groups of experts.
(2) The collection of artificial expert systems.
(3) Knowledge base.

Trade-off supporting subsystem supports the trade-offs between the achievement of qualitative goals and the opportunity costs.

Data base subsystem. The data base contains all data required for the models and graphs, namely: (1) Set of decision variables. (2) Set of constraints. (3) Coefficients in the objective function. (4) Technological coefficients. (5) Resources of right hand side. (6) Forecast of demand for each product. (7) Interval of possible changes for each a_{ij}, b_i, c_j. (8) Relative market share of each product – for decision maker and competitors for BCG matrices. (9) Industry growth rate of each product – for decision maker and competitors for BCG matrices. (10) Sales of each product – for decision maker and competitors for BCG matrices. (11) Market attractiveness of each product – weighted sum for GE matrices. (12) Product strength of each product – weighted sum for GE matrices. (13) Market size of each product. (14) Sales history of decision maker's products – for trend graphs for decision maker products and competitor's products. (15) History of market size of all products.

Dialogue subsystem. It suggests user-friendly features among algorithm, computer and user.

ILLUSTRATION BY EXAMPLE

To illustrate the procedure, the following example is presented. Let's suppose a farm that produces 5 crops and realises 5 other activities in order to maximise its profit in a competitive market and owner (Mr Smith) wants to establish a plan to maximise his farm's profit from the long-term point of view. Based on his own experience, he knows the existence of conflicts between long-term profits and short-term profits. Therefore, his major concern is, how to strike a balance between long-term profits and short-term profits.

To apply the procedure and to follow the algorithm above, we need the internal information about Smith own farm's available resources as well as the external information about the market and the farmer's competitors. To follow formulas (1) – (4) Mr Smith formulated linear programming model of his farm as follows:

(1) The set of restricting conditions $\mathbf{Ax}\ \{\leq, =, \geq\}\ \mathbf{b}$

1. $x_1 + x_2 + x_3 + x_4 + x_5 \leq 200$
2. $x_1 - x_2 - x_3 - 2x_4 \leq 0$
3. $-0.4x_1 + 10x_7 + 10x_8 \leq 0$
4. $-24x_1 + x_9 + x_{10} \leq 0$

5. $x_7 \leq 5$
6. $x_9 \geq 0$
7. $-7.5x_3 - 9.52x_4 - 6.05x_5 - 0.08x_6 + 3.1x_8 - 2.3x_{10} \leq 0$
8. $-0.6x_3 - 0.52x_4 - 0.14x_5 - 0.007x_6 + 0.36x_8 - x_{10} \leq 0$
9. $-3.48x_3 - 5.53x_4 - 3.8x_5 - 0.07x_6 + 2.3x_8 - 1.61x_{10} \leq 0$
10. $-30x_3 + 3.5x_6 \leq 0$
11. $-35x_3 + 4x_6 \leq 0$
12. $x_8 \leq 10$

(2) The set of qualitative constraints $q_g\ (\mathbf{x} \mid \mathbf{i}) \cup q_c\ (\mathbf{x} \mid \mathbf{i})$

13. $q_g\ (x_j \mid i_j) \cup q_c\ (x_j \mid i_j),\ j = 1, 2, \ldots, 10$

(3) Nonnegativity constraints $\mathbf{x} \geq \mathbf{0}$

14. $x_j \geq 0,\ j = 1, 2, \ldots, 10.$

(4) Objective function $z = \{\mathbf{cx}, q_g; q_c\} \rightarrow \max$

15. $z = \{12.5x_5 + 20x_6 + 1.2x_7 + 4.5x_9;\ q_g\ (x_j \mid i_j) \cup q_c\ (x_j \mid i_j)\} \rightarrow \max$

Variables $x_1, \ldots, x_{10}$ describe the optimal levels of farm products.

The objective 15 describes the aim of the short-term plan in terms of profit subject to qualitative human decisions concerning goal and restrictions, the constraints 1–12 describe limitations of available resources: production, capacity, capital budget, etc. The constraints 13 are associated with the judgement functions for the qualitative goals of long-term profits and the associated strategic constraints respectively.

The notation i_j means the additional information about the market and competitors of the product x_j, $j = 1, 2, \ldots, 10$. The constraints 13 imply that each product should contribute to the farm for the maximisation of long-term profits and focuses also external influences concerning restricting conditions, such as the regulations of government and the reaction of competitors.

As the first step of the procedure, the quantitative feasible solution of the model linear programming 1–15 was computed to maximise short-term profit. The optimal feasible solution and corresponding opportunity costs (dual values) is presented as follows:

Variable	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Optimal value	125.0	16.36	2.91	2.85	52.8	0.0	5.0	25.0	392.8	0.0
Opportunity cost	0.0	0.0	0.0	0.0	0.0	74.75	0.0	0.0	0.0	4.5

The next steps focus constraints 13 and follow the algorithms. To evaluate the solution from the long-term profit's point of view, the quantitatively optimal solution is displayed in BCG matrices, GE matrices and trend graphs. To draw matrices and graph Mr Smith can use the help of a consulting firm, available information and opinion of his neighbours. We shall illustrate the case of the product x_6. The optimal solution does not recommend to produce this product but it has a good position on the market. BCG and GE Matrices and Trend Graph for variable x_6 illustrate this situation in Figs. 2, 3 and 4. Mr Smith used following rules to obtain conclusion:

(1) If the decision maker's product is located in the north-east area of the BCG matrix, the product is a question mark product.

(2) If the competitor's product is positioned in the north-west area in the competitor's BCG matrix, the competitor product is a star product.

(3) If the product is positioned in the north-west area on the GE matrix, the product is a strong product.

(4) If the growth rate of the product is less than the market growth rate, the product is losing its market share.

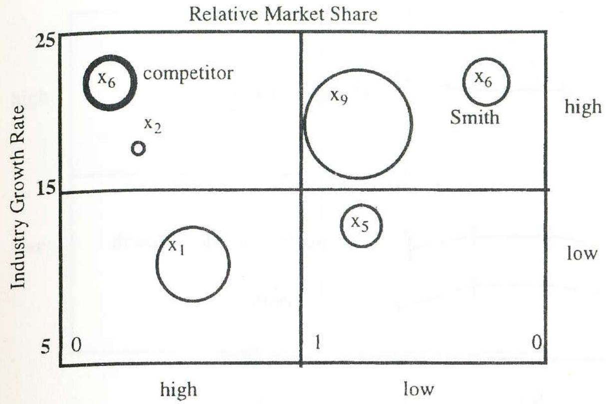
(5) If the growth rate of the competitor's product is less than market growth rate, than increasing market share is relatively easy.

(6) If the product is a question mark product and (i) increasing market share is relatively easy, (ii) the competitor's product is a star product, then the product is a potential star product.

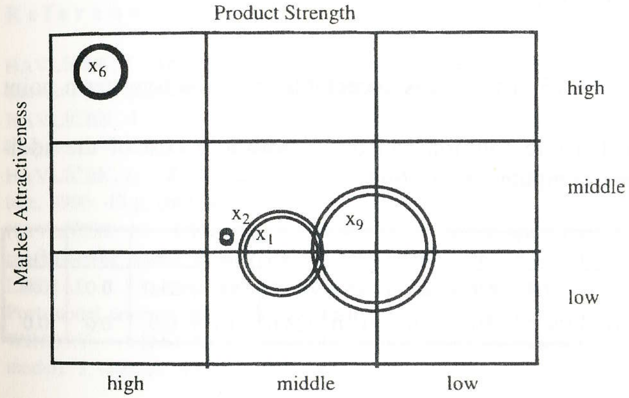
(7) If the product is potential star product and (i) increasing market share is relatively easy, (ii) the product is losing the market share, then augment production of this question mark product to increase its market share.

In this example we assume for x_6 that the growth rate of 10% is the subjective threshold of a higher growth rate*. In graph on Fig. 2 the product x_6 is positioned in the north-east area in BCG matrix. The product x_6 is thus question mark product. In graph on Fig. 3 the product x_6 is positioned in the north-west area in the EG matrix. The product x_6 is thus a *strong product* owing to the high market attractiveness and intensive product strength. Fig. 4 shows the growth rate of the competitor's product and the market growth rate for years 1991–1995. The competitor's growth rate is less than the market growth rate and thus increasing market share is *relatively easy*; competitor's product is a star product. The competitor is *losing its market share*. Mr Smith's market share of his product is also *decreasing*. Mr Smith can conclude: the product x_6 has significant potential to be a star product in the

* In the Czech Republic agriculture is in the transition process and the subjective measure of a high growth rate is much lower than in the western countries where it would be about 20%.



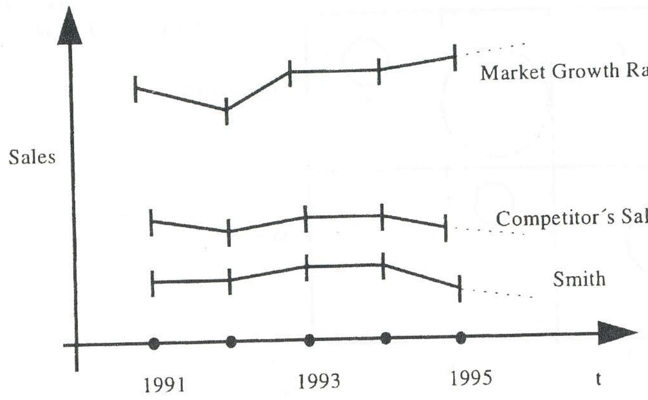
2. BCG matrix for Smith's product x_6



3. GE matrix for competitor's product x_6

future. Since the main competitor's product is losing market share, it is relatively easy to increase the market share of x_6 unless there is a hidden trap. If the product x_6 is not produced, the market share will be lost and that is not desirable from the long-term perspective. Mr. Smith should increase the market share of x_6 by increasing production.

Analysing the final simplex table, the production of x_6 can be increased from $x_6 = 0$ to $x_6 = 21$. However, increasing production of x_6 will influence the maximal profit which will be decline. Decreasing of profit is described by corresponding opportunity cost to x_6 which is equal to 74.75 per unit of x_6. If 21 products x_6 is added to the new solution, profit will be less and equal



4. Trend graph of competitor for product x_6

to 74.75 x 21, e.g. 1569.75. The loss is acceptable from the long-term point of view.

The new optimal feasible solution and the opportunity cost of the additional constraint were computed as follows:

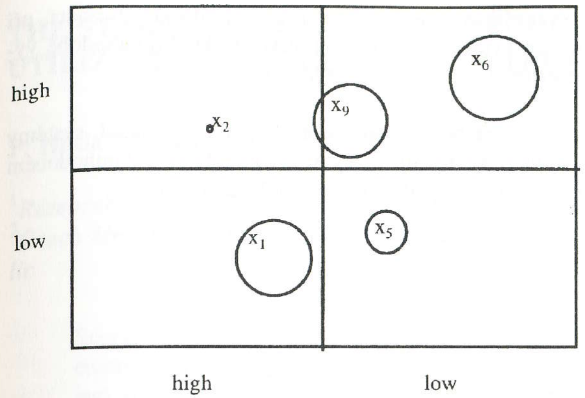
Variable	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
Optimal value	119.7	32.0	2.9	2.8	42.8	21.0	5.0	25.0	0.0	0.0
Opportunity cost	0.0	126.0	0.0	0.0	213.0	126.0	0.0	0.0	0.0	0.0

The product x_6 replaces the less attractive products.

The new BCG matrix is shown in Fig. 5. Mr Smith can continue the same procedure until he is satisfied for the product x_1 and repeated similarly the procedure for the other products and activities $x_2, x_3, \ldots, x_{10}$.

CONCLUSION

The post model analysis in the example above uses dual values in the optimal solution of the extended sales mix problem of linear programming. The judgement function utilises the additional information not included in the quantitative model combining graphical display, knowledge of experts and inquiry to the data base.



5. New BCG matrix for Smith's product x_6

References

HAVLÍČEK, J.: An interactive goal programming. In: Proc. Conf. Agrar Perspectives I, Czech University of Agriculture, Prague, 1992: 339-346.

HAVLÍČEK, J.: Decision support systems. Methodological study. Ministry of Agriculture, Prague, 1994. 34 p. (in Czech).

HAVLÍČEK, J. – ZÍSKAL, J.: Decision making if soft systems. Czech University of Agriculture, 1996. 46 p. (in Czech).

HAVLÍČEK, J. – DUŠEK, A. – ROUS, J.: Dynamic implementation of agricultural linear programming model. Zeměd. Ekon., 1973: 57–68 (in Czech).

LEE JAE, K.: Solving semi-structured problems and the design of decision support systems: Post-model analysis approach. University of Pennsylvania Press, 1985.

WIND, Y. – VIJAJ, M. – SWIRE, J. D.: An empirical comparison of standardised portfolio models. J. Market., *47*, 1983: 89–99.

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HAVLÍČEK, J. (Česká zemědělská univerzita, Provozně ekonomická fakulta, Praha, Česká republika):

Kvalitativní lineární programování založené na postoptimalizační analýze.

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Kvalitativní lineární programování se využívá v systému podpory rozhodování v měkkém systému. Strategie dlouhodobého plánování se opírá o informace, které jsou odvozeny (i) z modelu systému s krátkým časovým horizontem, (ii) z analýzy vývoje trhu v konkurenčním prostředí (BCG, GE matice a vývojové grafy), (iii) z rozboru výsledků primárního a duálního modelu lineárního programování (duální hodnoty, stínové ceny, transformovaná matice koeficientů lineárního modelu). Kvalitativní podmínky lze tak kvantifikovat nezávisle pomocí tří zdrojů informací, při analýze informací (ii) a (iii) lze využít expertů. Ilustrativní příklad dlouhodobé výrobní strategie farmy demonstruje navržený algoritmus řešení.

kvalitativní uvažování; model lineárního programování; kvalitativní odhad; systémy pro podporu rozhodování; strategické řízení; zisky v krátkodobém a dlouhodobém horizontu; BCG a EG matice; vývojové grafy; postoptimalizační analýza

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