FOREST PRODUCTION STEERING TO SUSTAINED YIELD UNDER RISK CONDITION BY INTERACTIVE LINEAR PROGRAMMING TASK CONSTRUCTION*

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The base of the presented tasks is the theory of normal forest, newly stochastically formulated by Kouba in 1969-1983, based on the theory of random processes, especially of homogenous, regular Markov chains. The purpose of this formulation was inclusion of interference, abiotic and biotic and anthropogenic one, which cause destruction (decrease) of the original forest area and present the main risk of forestry, in general of forest growth and development. Optimum management of forest production (of growth and age structure) consists in detection of the development path, which provides the closest possible conversion to the stochastically defined normal forest, with regard to a suitable criterion function. Optimum management steps have to conform to restrictions of decision-making space, given by probabilities of calamities by age classes (stages) and arbitrary-determined shares of maximum felling area of the appropriate age class. These problems lead to rather large linear programming tasks (hundreds, even thousands of equations and inequalities, thousands and tens of thousands of positive coefficients). It was necessary to elaborate programs for automated creation of such tasks that allow interactive use (Zahradník, Křepela, 2004). Creation of the above-mentioned task is based on the Kouba's previous works from the late 1980s devoted to estimation of calamities probabilities, based on the reliability theory, whose further development led to construction of forest life table (Kouba, 2002) and further use of the regular and absorption Markov chains. Thus, it is possible to open these methods up to practical use.

stochastical normal forest; conversion to normal forest; Markov chain; theory of reliability; linear programming

INTRODUCTION

Risk in forestry and its industries is represented above all specific, harmful factors naturally existing in the nature. The most important of them are the abiotic causes by wind, snow, ice frost, drought, fires, biotic causes by insects, wild animals, etc., and at the last anthropogenic causes as an impact of not always well judged development of human civilisation and also criminal acts (arsonism, and others). Apart from those, in forestry there are economic risks, common in all economic branches.

Specific forestry risks are manifested above all by threat to life of forest trees, forest stands, and forest cultures and ecosystems, generally. Risks to forest life can be considered from three aspects: a single tree, forest stand, forestry enterprise or the whole forest complex. Risk to forest life increases with its age and is specific to the forest type or single forest tree species. The risk can be generally expressed as the probability of life or death at a certain age stemming from the number of trees in a certain area or on the size of a forest that lived to a certain age or a size of forest destroyed at a certain age. The forest life risk can be expressed by constituting forest life tables, a forestry analogy of life tables for human population known from the demography and life insurance. Methods of constructing such table as one principle of forest property insurance were showed by Kouba (2002).

There are two ways to deal with the risk from the economic point of view:

1. Inclusion of specific forestry risk into forestry management and financial calculations and derivation of respective forest-economic consequences.
2. Transfer of forest risk to another institution - i.e. above all to insurance or re-insurance companies, for a certain price, of course, for insurance tax.

Application of the respective approaches depends on a particular situation. Both approaches assume knowledge of behaviour and magnitude of respective processes that can be expressed by application of specific methods of forest production mathematics and general financial and insurance mathematics stemming from above all probability theory methods and related special disciplines like theory of reliability, theory of catastrophes, operation research, cybernetics, theory of artificial intelligence and other methods.

One of the important tasks of forest management is adjustment of the present age structure of forest stands to the normal (or stationary) age structure. This adjustment can be carried out by various ways. The optimal way can be found by means of linear programming. Salvage cutting in each age class (stages) form the lower boundary, while legal rules form the upper boundary of the decision-making area, where it is possible to choose planned felling with the aim of rapid conversion to the

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normal age structure. The objective function is, for example, the sum of cutting during the conversion process. Such linear programming problem is very extensive in common situations (hundreds, even thousands of equations and inequalities, thousands and tens of thousands of positive coefficients). It is inconceivable to construct this problem manually. In order to tackle this difficulty, we have developed a new part of software system, called ForeProb (referred also in Nabuurs, Päivinen, 1996) after a consistent research during a period of several years. The author of the software program, including the part presented here is Zahradník (Zahradník, Křepela, 2004). The software program has three inputs: probabilistic distribution of salvage cutting (we suppose they follow Weibull's distribution law), legal rules for maximal cutting and initial (present) age structure. The result is automatic formulation of linear programming problem in form, which can be easily exported into Mathematical software of the Wolfram Company. Then, the optimal solution is found in this program. The result consists in optimal sizes of cutting in individual age classes for each step of the balancing process. Based on the knowledge of these sizes, the time course of the age structure development, the sizes of the growing stock, growth and other quantities can be calculated afterwards. The program provides a very fast construction of a great number of variants for the mentioned linear programming problems and so allows gaining required experience for further formalisation of the problem solution.

METHODS

Stationary distribution of age classes

The traditional normal forest theory considers even distribution of age classes as optimum. However, this theory does not take into account the occurrence of salvage cutting. In fact, salvage cutting represents a major part of total cutting. If a part of the forest stands is destroyed before the planned main cutting age is achieved, it is obvious that the stationary distribution of age classes cannot be even as younger stands must have higher representation than older stands. A method for determining the stationary age structure (with respect to the salvage cuttings) has been developed by Kouba (1974, 1977). In this method, the forest age structure is expressed in terms of a regular Markov chain. The probabilities of occurrence of the salvage cutting in individual age classes as well as the stationary age structure (equivalent with the age structure determined by the first method) can be computed using the theory of renewal (reliability) (Kouba, 1989).

The age of the stand, where salvage cutting occurs, is a random variable that can be described by the distribution function $F(t)$ of some probability distribution. In addition, it is convenient to work with a complement of the $F(t)$ function, called the function of reliability:

$$R(t) = 1 - F(t) \tag{1}$$

So this function determines the probability that stand of the age t has not been destroyed by a disaster.

Many harmful factors only occur in certain life periods of the stands. Every such factor, however, only destroys a certain part of the total forest area. Then the reliability functions of partial harmful factors $R_i'(t)$ are modified to the following form

$$R_i'(t) = c_i R_i(t) + (1 - c_i) \tag{2}$$

where: c_i – part of the area destroyed by i-th factor

The aggregate reliability function is

$$R(t) = \prod_{i=1}^{k} R_i'(t) \tag{3}$$

where: k – number of harmful factors

This aggregate function $R(t)$ can be used to calculate the stationary distribution of age classes:

$$a_i = \frac{\int_{(i-1)r}^{ir} R(t)\,dt}{\int_0^{mr} R(t)\,dt}, \quad i = 1, 2, \ldots, m \tag{4}$$

where: m – number of age classes
r – time span of the age class (conventionally 10 or 20 years)

Subsequently, we can calculate the area proportions of stands destroyed in individual age classes:

$$p_i = 1 - \frac{\int_{ir}^{(i+1)r} R(t)\,dt}{\int_{(i-1)r}^{ir} R(t)\,dt}, \quad i = 1, 2, \ldots, m \tag{5}$$

We suppose that partial harmful factors follow Weibull's distribution law, thus

$$R_i'(t) = c_i \exp(-\lambda_i t^{\alpha_i}) + (1 - c_i), \alpha_i > 0, \ \lambda_i > 0 \tag{6}$$

The parameters α_i and λ_i can be computed by using estimates $R_i(t_1)$ and $R_i(t_2)$at two convenient time moments t_1 and t_2:

$$\alpha_i = \frac{\ln\left(\frac{\ln R_i(t_1)}{\ln R_i(t_2)}\right)}{\ln\left(\frac{t_1}{t_2}\right)} \tag{7}$$

$$\lambda_i = \frac{-\ln R_i(t_1)}{t_1^{\alpha_i}} \tag{8}$$

In detail, see Kouba (1989, 1991, 2002).

Linear programming solution

The task of conversion to classical normal (even) age structure (without considering calamities) was tackled by Nautiyal and Pearse (1967). The topic of this article deals with solution of optimum steering of the conversion process from the current forest towards the normal one defined stochastically regarding the area distribution of salvage cuttings and with respect to the effect of salvage cuttings during this conversion (Kouba, 1989).

It is obvious that the initial age structure can be adjusted to the target structure only with a suitably selected size of main cutting in corresponding age classes. The lower limit of cutting proportions in individual age classes results from area proportions of salvage cutting in these classes. The last age class, which is always harvested completely, represents an exception to the rule. The upper limit of cutting actually results from the legislation. In age classes where planned main cutting is not permitted yet, it is the same as the lower limit. In higher classes, the whole area of the class can be harvested. The objective function that we want to maximise is the total volume of all cutting within the process of conversion to the stochastically defined normal forest.

Now let us introduce some symbols. Let us assume that we have forest stands with m age classes and that we want the conversion process to have n steps. The length of one step is the same as the span of the age class. Let us mark the area proportion of cutting in the j-th step and i-th age class as x_i^j. The area proportion of the i-th age class at the beginning of the j-th step will be marked as v_i^j. Therefore, the v_i^1 elements represent the initial age structure. The area proportion of the i-th age class in the target structure will be a_i. By analogy, the l_i and u_i symbols indicate the lower and upper limit of the area proportion of cutting in the i-th age class.

To obtain limiting conditions for x_i^j, let us first determine what size of the v_i^j area can be expected. Then, x_i^j must satisfy the following inequalities:

$$l_i v_i^j \le x_i^j \le u_i v_i^j,\ i = 1, 2, \dots, m,\ j = 1, 2, \dots, n \qquad (9)$$

There are constraints of two types.

Constraints of type I.

If $j \le i \le m$, it is obvious that the v_i^j area is a remainder of the initial v_{i-j+1}^1 area. Within the previous $j - 1$ steps of the conversing process,

$$\sum_{k=1}^{j-1} x_{i-j+k}^k,\ j = 2, 3, \dots, n,\ i = j,\ j+1, \dots, m$$

of the area was cut out gradually. So the constraints for x_i^j are

$$\left.\begin{aligned} x_i^1 &\ge l_i v_i^1 \\ x_i^1 &\le u_i v_i^1 \end{aligned}\right\} j = 1,\ i = 1, 2, \dots, m \qquad (10)$$

and

$$\left.\begin{aligned} x_i^j &\ge l_i \left(v_{i-j+1}^1 - \sum_{k=1}^{j-1} x_{i-j+k}^k \right) \\ x_i^j &\le u_i \left(v_{i-j+1}^1 - \sum_{k=1}^{j-1} x_{i-j+k}^k \right) \end{aligned}\right\} j = 2, 3, \dots, n,\ i = j,\ j+1, \dots, m \qquad (11)$$

Constraints of type II.

If $i < j$, the v_i^j area is a remainder of the v_1^{j-i+1} area. At the same time this area is the result of the sum of all areas cut in the $(j - i)$-th step:

$$v_1^{j-i+1} = \sum_{k=1}^{m} x_k^{j-i},\ j = 2, 3, \dots, n,\ i = 1, 2, \dots, \min(j-1, m)$$

Within the past $i - 1$ steps the following amount was cut from this area:

$$\sum_{k=1}^{i-1} x_k^{k+j-i},\ j = 3, 4, \dots, n,\ i = 2, 3, \dots, \min(j-1, m)$$

thus,

$$\left.\begin{aligned} x_1^j &\ge l_1 \sum_{k=1}^{m} x_k^{j-1} \\ x_1^j &\le u_1 \sum_{k=1}^{m} x_k^{j-1} \end{aligned}\right\} j = 2, 3, \dots, n,\ i = 1 \qquad (12)$$

and

$$\left.\begin{aligned} x_i^j &\ge l_i \left(\sum_{k=1}^{m} x_k^{j-i} - \sum_{k=1}^{i-1} x_k^{k+j-i} \right) \\ x_i^j &\le u_i \left(\sum_{k=1}^{m} x_k^{j-i} - \sum_{k=1}^{i-1} x_k^{k+j-i} \right) \end{aligned}\right\} j = 3, 4, \dots, n,\ i = 2, 3, \dots, \min(j-1, m) \qquad (13)$$

Finally, we must make sure that after the last step of conversion period the areas of all age classes have the required target size ($v_i^{n+1} = a_i$). There are conditions of two types again.

Constraints of type III.

If $n \ge m$ or $i \le n$, the v_i^{n+1} area is the result of the sum of all areas cut in the $(n - i + 1)$-th step, reduced by area:

$$\sum_{k=1}^{i-1} x_k^{k+n-i+1},\ i = 2, 3, \dots, m$$

thus,

$$\sum_{k=1}^{m} x_k^n = a_i,\ i = 1 \qquad (14)$$

and

$$\sum_{k=1}^{m} x_k^{n-i+1} - \sum_{k=1}^{i-1} x_k^{k+n-i+1} = a_i,\ i = 2, 3, \dots, m \qquad (15)$$

Constraints of type IV.

If $n < m$ and $i > n$, the v_i^{n+1} area is a remainder of the initial v_{i-n}^1 area. Within the previous n steps was cut area

$$\sum_{k=1}^{n} x_{k+i-n-1}^k,\ i = n+1, n+2, \dots, m$$

hence,

$$v_{i-n}^1 - \sum_{k=1}^{n} x_{k+i-n-1}^k = a_i,\ i = n+1, n+2, \dots, m \qquad (16)$$

The objective function has the following form:

$$\sum_{j=1}^{n} \sum_{i=1}^{m} z_i x_i^j \to \max \qquad (17)$$

where z_i is the size of hectare growing stock in the i-th age class.

RESULTS AND DISCUSSION

To illustrate the use of the proposed procedure, we present an example. Let us suppose that we have a set of spruce forest stands with 17 age classes (with the time span of 10 years) and with site index 28 (Halaj, Petráš, 1998). We assume there are four harmful factors. Their list and parameters of corresponding reliability functions R_i' (we use estimates by Kouba (1989)) are given in Table 1.

The target distribution of age classes was computed using formula (4). We reduce the number of target age classes to 11, because total production of stands with site index 28 culminates at the age of 102 years. The lower bound of decision-making area is given by area proportions of stands destroyed in individual age classes. These proportions are computed by formula (5). We allow to cut out whole area in forest stands older than 80 years. Initial age classes distribution in this example is based on the National Forest Inventory 2000. All these parameters are summarised in Table 2.

We want to normalise the age structure as soon as possible. The smallest number of steps necessary to normalise the age structure in our example is 11. In Figs 1 and 2, we present the development of the basic mensurational quantities characterizing the process of optimal felling control towards the targeted sustained yield. Apart from that, it is possible to print or graphically present the development of the area proportions of age stages in all particular computing steps, and area proportions of random and planned cutting. The volume of the data obtained is rather extensive and is not introduced here in order to save space.

Fig. 3 shows the results of further examples computations expressed generally as trajectories of the relationship development between the total cut and total increment, which is based on analogous expression for a regular Markov chain (Kouba, 1977). This trajectory starts from the origin stage (relationship between the cut and increment in first step of the conversion process) and gradually approaches the stationary stage (relationship between the cut and increment after last step of the conversion process) where the cutting logically equals the increment. Such demonstration also shows that the relationships between the increment and possible cut are not really that immediate and linear as the foresters sometimes assume. There are visualised the results for three examples. Forest stands in all examples have site index 28. They also have the same stationary age classes distribution. The examples differ in initial age structure and in length of the conversion process. In first example (labelled as "28"), the initial age classes distribution is

Table 1. The list of harmful factors and parameter estimates of corresponding reliability functions

Factor	α	λ	c
Destruction of young plantations	1.0675	1.0986	0.45
Snow	4.2002	$4.2778.10^{-8}$	0.2
Wind	3.9332	$1.4334.10^{-9}$	1
Undefined random damage	1	0.0005	1

Table 2. Input parameters for formulation of linear programming problem

Age class	Initial age structure	Target age structure	Lower bound	Upper bound
1	0.088	0.114	0.074	0.074
2	0.080	0.106	0.011	0.011
3	0.074	0.104	0.025	0.025
4	0.083	0.102	0.046	0.046
5	0.066	0.097	0.066	0.066
6	0.083	0.091	0.068	0.068
7	0.104	0.084	0.051	0.051
8	0.087	0.080	0.036	0.036
9	0.088	0.077	0.036	1
10	0.085	0.074	0.046	1
11	0.064	0.071	0.058	1
12	0.041	0	0.073	1
13	0.025	0	0.090	1
14	0.014	0	0.110	1
15	0.008	0	0.132	1
16	0.004	0	0.156	1
17	0.006	0	1	1

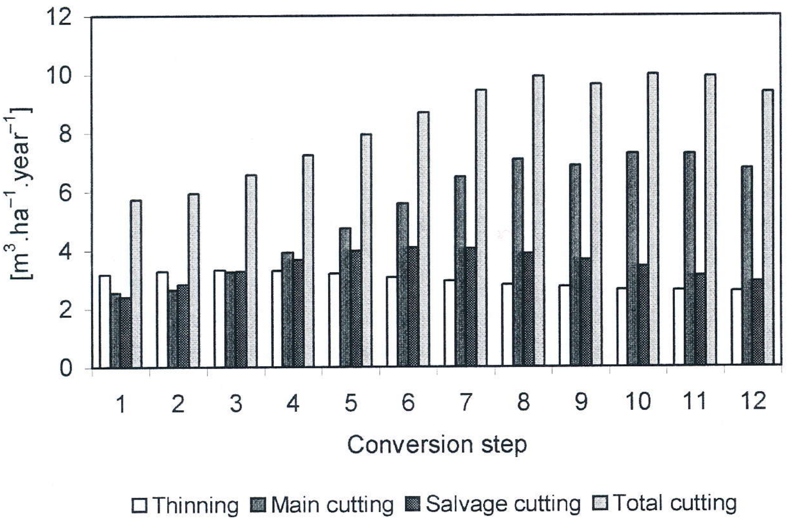

Fig. 1. Development of thinning, salvage cutting and total cutting

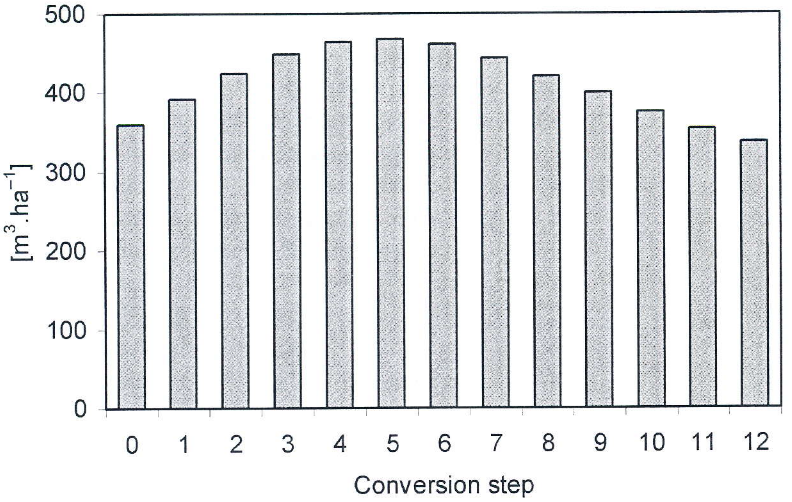

Fig. 2. Development of growing stock

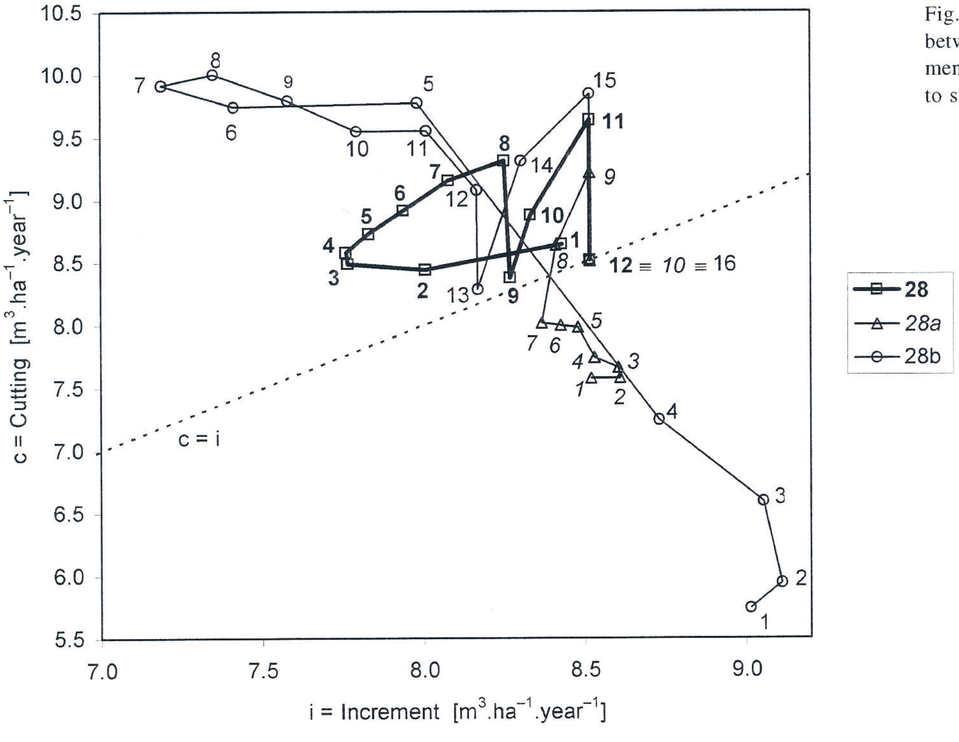

Fig. 3. Development of relationship between the total cut and total increment during the conversion process to sustained yield

based on the National Forest Inventory 2000 and the conversion process has 11 steps. In example "28b", the initial age structure is the same, but the conversion process has 15 steps. Example "28a" represents the calculation for the initial age structure from year 1920, when the forests in the Czech Republic had a lower mean age resulting from shorter rotation periods used then. The conversion process has 9 steps. The conversion processes in these examples converge to the same point, because the stationary age classes distributions are also the same.

The results have so far showed that the study of the processes regulating the forest production and development using mathematical methods is of a considerable theoretical and practical importance. These methods allow us to deepen our knowledge and experience, they allow us to understand profoundly and consider better what such principal concepts like the growing stock, the increment, and the cutting volume really represent and what their relationships may be. They provide for better consideration of the risk rate in forest production and assessment of its impact on the forestry-wood-processing sector's economic effect.

In the end, we would like to apologise to our readers that the paper length limit did not allow us to present all references on this issue. More information together with a certain review can be found in works of Nabuurs, Paivinen (1996), Zadnik-Stirn (1999), von Gadow, Puumalainen (2000) and works of Kouba (1977, 1989, 1991, 2002).

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KOUBA, J. – ZAHRADNÍK, D. (Česká zemědělská univerzita, Fakulta lesnická a environmentální, Praha, Česká republika):

Řízení procesu vyrovnání k trvale udržitelnému lesu při úvaze rizika pomocí interaktivně konstruovaných úloh lineárního programování.

Scientia Agric. Bohem., *35*, 2004: 140–146.

Základem prezentovaných úloh je teorie normálního lesa formulovaná nově autorem v letech 1969–1983, na základě teorie náhodných procesů, speciálně pak homogenních, regulárních Markovových řetězců. Smyslem této formulace bylo zahrnutí rušivých vlivů, abiotických, biotických i antropogenních, způsobujících zničení (úbytek) výchozí plochy lesa a představujících tak hlavní riziko lesního hospodářství, obecně růstu a vývoje lesů. Optimální řízení produkce (růstu a věkové struktury) lesů pak spočívá v nalezení takové trajektorie jejího vývoje, která umožňuje blížit se tomuto stochasticky definovanému normálnímu lesu s ohledem na vhodnou kriteriální funkci. Optimální kroky řízení musí pak vyhovovat omezením prostoru rozhodování, daným pravděpodobnostmi kalamit podle věkových stupňů a arbitrárně určenými podíly maximální těžební plochy v příslušném věkovém stupni. Takové problémy vedou k velmi rozsáhlým úlohám lineárního programování (stovky až tisíce rovnic a nerovností, tisíce až desetitisíce kladných koeficientů). Bylo třeba vypracovat programy pro automatizované vytvoření takových úloh, aby bylo možné s nimi pracovat interaktivně (Zahradník, Křepela, 2004). Vytvoření uvedené úlohy vychází z prací autora z konce 80. let, věnovaných odhadu pravděpodobností kalamit, na základě teorie spolehlivosti, jejichž další rozvinutí vedlo ke konstrukci tabulek života lesa (Kouba, 2002) a k dalšímu využití regulárních a absorpčních Markovových řetězců. Tímto způsobem je možné zpřístupnit tyto metody praktickému využití.

stochasticky normální les; přechod na normální les; Markovův řetězec; teorie spolehlivosti; lineární programování

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