REINFORCEMENT OF PLATE STRUCTURES BY EQUALLY LOADED BEARING LINE ELEMENTS

R. Novotný

Czech University of Agriculture, Faculty of Forestry and Environment, Prague, Czech Republic

The paper intends to give a practical and simple instruction for the distribution of a finite number of load-bearing linear members in a plane or cylindrical structure subjected to a general continuous load of cylindrical or translational distribution with surface lines or parallels of conforming orientation to assure that the beams are exposed to loads of equal intensity. The paper shows (both analytically and graphically) in particular the location of these load-bearing members with reference to linearly increasing loads which is of extraordinary importance in engineering practice.

hydrostatic pressure; earth pressure; retaining walls with horizontal beams; basement walls subjected to horizontal loads; vertical cylindrical (particularly circular) shells with liquid or loose solid fill stiffened with horizontal members (particularly rings)

INTRODUCTION

In engineering practice it is often necessary to distribute stiffening or load-bearing linear members of a two-dimensional structure so as to assure their uniform (i.e. equal or at least approximately equal) exposure to loads. This problem can be solved "almost always" graphically (and usually "adequately" also analytically), if the three-dimensional loading diagram of the two dimensional structure is represented by a "reasonable" cylindrical or translational or rotationally symmetrical surface with surface lines or parallels which are parallel with the load-bearing members in case of a plane structure and in parallel position in case of a cylindrical wall or a hip (Fig. 1). Consequently, it is assumed that the two-dimensional structure is of plane (wall or plate) or cylindrical type.

The above-mentioned structural requirement arises either from the fact that only identical (with reference to both the quality of material and the cross section dimensions and shape) load-bearing members are available, what results in a more or less natural assumption of the constant thickness of the load-bearing surface or because it is not permissible for some reason or other to alter the thickness of the plane structure with built-in stiffeners (e.g. in accordance with the depth below the level of the compressive medium, etc.). Let us note that for an entirely general (i.e. non-cylindrical or possibly non-translational or rotationally symmetrical) load applied to the plane structure (the less for the structure with generally defined centre line) with linear stiffeners the required solution does not exist, as a rule.

The outlined problem may apply, e.g. to beam floors loaded with loose material distributed in accordance with introduced assumptions, reinforced concrete basement or retaining walls (whether stiffened with horizontal beams or ribs or not) acting in horizontal direction and exposed to earth pressure, sluice gates and mobile panel weirs resisting hydrostatic pressure, vertical circular cylindrical or generally cylindrical tanks filled with some liquid or loose solid medium, etc.

As the solution of the problem is entirely analogous both for plane and cylindrical linearly stiffened load-bearing surfaces, we shall concentrate on the derivation of the required rules and their interpretation for the simpler plane formation.

MATERIAL AND METHODS

The solution of the mentioned engineering problem is based on the mathematical principles of integral calculus (in particular the properties of summation curves, which are connected with definite and indefinite integral of generally defined load function). From this mathematical analysis follows – for "triangulary" defined load function – also well known geometrical solution based on Euklides-Thalet theorem.

Determination of transverse intervals for identical intensites of load applied to linear members in longitudinal direction and location of stringers

Let us consider a plane load-bearing formation, such as a plate, stiffened with stringers (straight linear beams), loaded in its whole with l by a cylindrical continuous load with surface lines in stringer direction. Let the section perpendicular to the stringers of the loading cylinder be defined by a continuous load function $q(x)$ – see Fig. 2. The objective is to assure that every stringer should support an identical part of the load or, in other words, to find the limits x_j of finite intervals within which the trapezes with one (generally) curved side

$$\int_0^{x_1} q(x)\cdot dx = \ldots \int_{x_1}^{x_{j+1}} q(x)\cdot dx = \int_{x_{j+1}}^{x_{j+2}} q(x)\cdot dx = \ldots =$$

$$= \int_{x_{n-1}}^{x_n} q(x)\cdot dx = \frac{1}{n}\cdot\int_0^1 q(x)\cdot dx$$

are of equal area with the understanding that the individual longitudinal beams will be situated in their cen-

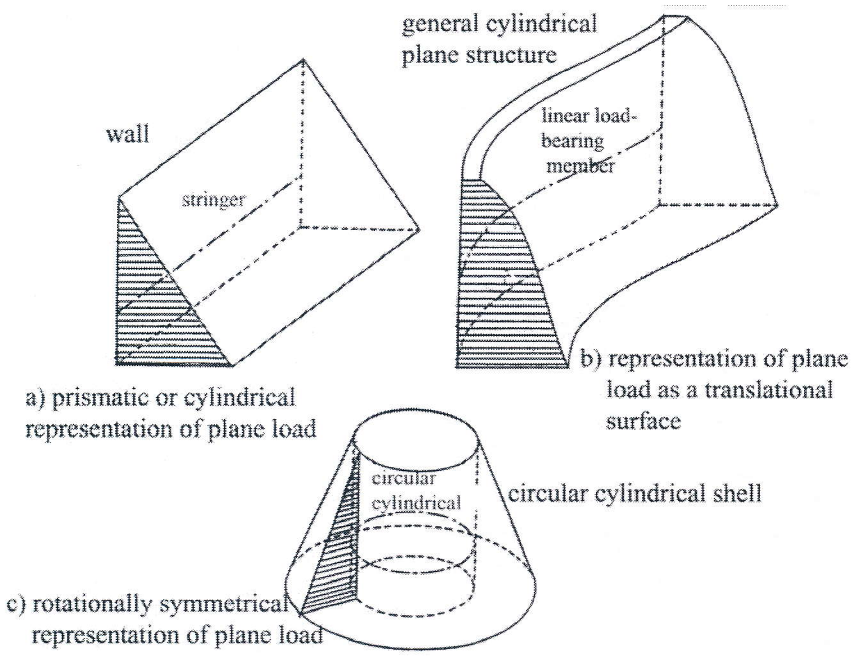

Fig. 1. Character of examined plane structures with linear load-bearing members and the types of their all-surface loads (some typical examples)

troids, $j = 1, 2, 3, \dots, n$. In other words such x_j coordinates are sought as will satisfy the equivalent condition.

$$\int_0^{x_j} q(x)\cdot dx = \frac{j}{n}\cdot\int_0^{1} q(x)\cdot dx \tag{1}$$

while $\sum_{j=1}^{n}(x_j - x_{j-1}) = l$.

The positions of the total n centroids further arise, according to the Varignon theorem, from the relation

$$t_j = \frac{\int_{x_{j-1}}^{x_j} q(x)\cdot x\cdot dx}{\int_{x_{j-1}}^{x_j} q(x)\cdot dx}, \tag{2}$$

while $t_j \in (x_{j-1}; x_j)$ t_j and $x_0 = 0$. Let us note that the integral curve $Q(x) + C = \int q(x)\cdot dx + C$ in Fig. 1 has been so chosen as to pass through the origin O, i.e. to correspond with integration constant $C = 0$, as evidently $\frac{d\,[Q(x) + C]}{dx} = q(x)$.

Transverse intervals and location of equally loaded stringers for loads with linear intensity increase

Let us consider now a special "loading cylinder", see a prism in the base of which, perpendicular to the stringers, its direction line is defined by the load function $q(x) = q_0 + k\cdot x$ (see Fig. 3).

The application of the general condition (1) derived the above results easily in four-parameter quadratic equations (with q_0, k, l and n, which are usually chosen, as independent parameters) yielding successively n real solution.

$$x_j = \frac{\sqrt{q_0^2 + \frac{2\cdot k\cdot j}{n}\cdot\left(q_0\cdot l + \frac{1}{2}\cdot k\cdot l^2\right)} - q_0}{k}, \tag{3}$$

$j = 1, 2, 3, \dots, n$. Let us note that in case of hydrostatic problems the initial pressure may be represented e.g. by atmospheric pressure, when $k = 1$, while in case of earth pressures the initial horizontal pressure q_0 arises from additional ground loading by a certain vertical load different from zero and the parameter k plays the role of the lateral pressure coefficient (active, passive or at rest).

If we denominate now $q_j = q(x_j)$, we arrive easily to the relation

$$q_j = \sqrt{q_0^2 + \frac{2\cdot k\cdot j}{n}\cdot\left(q_0\cdot l + \frac{1}{2}\cdot k\cdot l^2\right)}. \tag{4}$$

Apart from that the condition (2) yields the formula

$$t_j = \frac{q_{j-1}\cdot(x_{j-1} + x_j) + (q_j - q_{j-1})\cdot\frac{1}{3}\cdot(x_{j-1} + 2\cdot x_j)}{(q_{j-1} + q_j)}, \tag{2a}$$

valid for the assumed "trapezoidal" load defined by the equation $q(x) = q_0 + k\cdot x$.

Let us concentrate now on the "triangular" load defined by the relation $q(x) = k\cdot x$ or let us assume that $q_0 = 0$. In that case Eq. (1) acquires the simple form of

$$\frac{1}{2}\cdot k\cdot x_j^2 = \frac{j}{2\cdot n}\cdot k\cdot l^2, \tag{1a}$$

which shows at first sight that it holds

$$x_j = l\cdot\sqrt{\frac{j}{n}}. \tag{3a}$$

The simple biparametric Eq. (3a) is a special case of Eq. (3). Let us note that during its derivation it was possible (due to the absence of q_0) to eliminate also the parameter k. For the sake of completeness, let us note

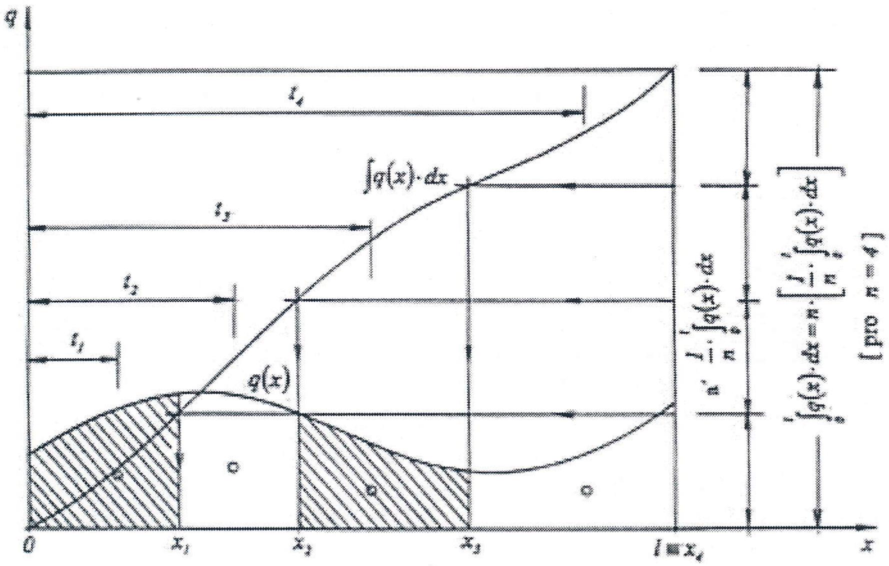

Fig. 2. Load $q(x)$ uniformly distributed to stringers for $n = 4$

that for the assumed "triangular" load Eq. (4) is converted into the simple equation

$$q_j = q(x_j) = k \cdot l \cdot \sqrt{\frac{j}{n}} = k \cdot x_j. \tag{4a}$$

Due to the simplicity of Eq. (3a) the break-up of the (transverse) interval l into individual sections $x_j - x_{j-1}$ under linear load $q = k \cdot x$, valid for all $x \in \langle 0; 1 \rangle$; so as to subject all stringers to equal load, can be tabulated (Table 1).

If we apply now Eq. (4a) to Eq. (2a) we come, after rearrangement, to the formula for the determination of n centroids t_j of trapezes of (generally) equal area $\frac{k \cdot l^2}{2 \cdot n}$, into which a rectangular triangle of the base l disintegrates. Thus we obtain "location" formulas of the type

$$\frac{t_j}{l} = \frac{x_{j-1}}{l} + \frac{\left(\frac{x_j}{l} - \frac{x_{j-1}}{l}\right) \cdot \left(2 \cdot \frac{x_j}{l} + \frac{x_{j-1}}{l}\right)}{3 \cdot \left(\frac{x_j}{l} + \frac{x_{j-1}}{l}\right)}. \tag{2b}$$

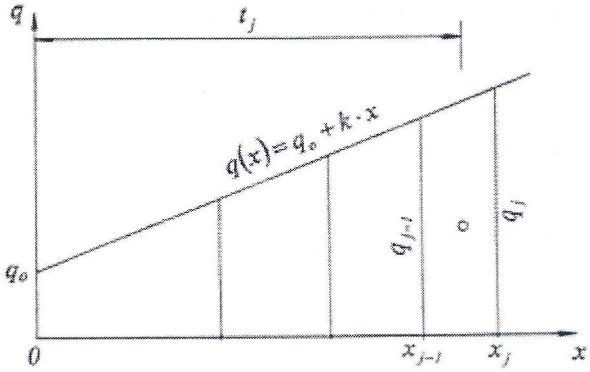

Fig. 3. Linearly increasing load $q(x) = q_0 + k.x$ uniformly distributed to stringers for $n = 4$

The alternative form (2b) can be used to advantage for the structuring of the above Table 1 into the tabulation of relative centroid coordinates $^{t}/_{l}$ for j = 1, 2,..., n = 10 (Table 2).

Table 1. Relative co-ordinates $^{x}/_{l}$ allocating to stringers equal load from transverse sections $\langle x_j - x_{j-1} \rangle$ for the load $q = k \cdot x$ applicable to all $x \in \langle 0; 1 \rangle$, j = 1, 2,..., n = 10

n	$^{x}/_{l}$									
	$^{x_1}/_{l}$	$^{x_2}/_{l}$	$^{x_3}/_{l}$	$^{x_4}/_{l}$	$^{x_5}/_{l}$	$^{x_6}/_{l}$	$^{x_7}/_{l}$	$^{x_8}/_{l}$	$^{x_9}/_{l}$	$^{x_{10}}/_{l}$
1	1									
2	0.7071	1								
3	0.5774	0.8165	1							
4	0.5000	0.7071	0.8660	1						
5	0.4472	0.6325	0.7746	0.8944	1					
6	0.4082	0.5774	0.7071	0.8165	0.9129	1				
7	0.3780	0.5345	0.6547	0.7559	0.8452	0.9258	1			
8	0.3536	0.5000	0.6124	0.7071	0.7906	0.8660	0.9354	1		
9	0.3333	0.4714	0.5774	0.6666	0.7454	0.8165	0.8819	0.9428	1	
10	0.3162	0.4472	0.5477	0.6325	0.7071	0.7746	0.8367	0.8944	0.9487	1

Table 2. Relative co-ordinates $^{t_j}/_{l}$ of centroids of sections $<x_j - x_{j-1}>$ under the circumstances of Table 1

n	$^{t_j}/_{l}$									
	$^{t_1}/_{l}$	$^{t_2}/_{l}$	$^{t_3}/_{l}$	$^{t_4}/_{l}$	$^{t_5}/_{l}$	$^{t_6}/_{l}$	$^{t_7}/_{l}$	$^{t_8}/_{l}$	$^{t_9}/_{l}$	$^{t_{10}}/_{l}$
1	0.667									
2	0.4714	0.862								
3	0.3849	0.7038	0.9113							
4	0.3333	0.6095	0.7892	0.935						
5	0.2981	0.5452	0.7059	0.8359	0.948					
6	0.2721	0.4976	0.6444	0.7631	0.8656	0.957				
7	0.2520	0.4607	0.5966	0.7065	0.8014	0.8861	0.963			
8	0.2357	0.4310	0.5581	0.6609	0.7496	0.8289	0.9011	0.968		
9	0.2222	0.4063	0.5262	0.6231	0.7067	0.7815	0.8496	0.9127	0.9717	
10	0.2108	0.3854	0.4991	0.5911	0.6705	0.7414	0.8060	0.8659	0.9218	0.975

CONCLUSION

The paper gives a detailed instruction (including an analytical substantiation) for the stiffening of some selected plane elements by a finite number of linear equally loaded beams under specially defined all-surface load. In accordance with the reality of the solution the instruction is limited to plane elements (plates and walls) or cylindrical plane elements with the proviso that load can be represented by a generally cylindrical surface with the base perpendicular to linear beams (stringers) or a general translational surface with "meridian" sections perpendicular to (generally not straight) linear beams (playing the role of parallels to the load-bearing surface).

The general progress of the solution of the problems of the above-mentioned type is based on the principles of integral calculus (in particular the properties of summation curves). The paper contains i.e. also the tabulation of the relative coordinates for the break-up of the interval with linear load function so that within their limits the linear beams concerned are loaded with equal intensity and contain also the tabulated relative co-ordinates of location of these beams for their total number from one to ten inclusive.

It follows from the context of the paper that the existence of the solution (apart from the explicitly defined geometry of the structure) will depend also on the stability of the load. Hence, the assumption of a certain fixed load function (in the "meridian" section). Therefore, any other than the initial (given) state would necessarily cause inequalities in the loading of firmly installed linear load-bearing members which was excluded from our considerations. We have considered the mutual spacing of these parallel load-bearing members significantly inferior to their lengths (spans) This condition is complied with the more, the higher the number of these load-bearing members for the given height of the plane structure. This points, inter alia, also to the considerably higher structural height of the beams in comparison with the structural thickness of the supported surface. Apart from that it is necessary to point out that the solved problem can find broad application both to the structures of one material (reinforced concrete, steel, timber) and in case of combined materials, composite structures (steel with reinforced concrete, steel with wood, etc.) which are less homogeneous. The outlined circumstances substantiate the structural dominance of beams as compared with the supported surface so that the neglect of their mutual interaction can be considered as "acceptable". On the other hand it is advisable to emphasize that the determination of the distribution of linear load-bearing members of unequal stiffness to assure that the given load be regularly distributed among them is a far more complicated (statically indeterminate) problem. This very fact substantiates the above applied assumption of the application of identical linear members, which is so frequent in engineering practice and has offered simultaneously a relatively easy solution presented in the paper.

The objective of the paper was not only to show some theoretical relations, but also – and particularly – to provide certain practical aids (see the appropriate formulas and Tables 1 and 2 in Parts 2 and 3 of the paper) for structural engineers and designers useful for the design of plane structural members some examples of which were mentioned in the introduction and for the solution of technical problems of analogous type.

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Vyztužování plošných konstrukcí stejně namáhanými liniovými nosnými prvky.

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V běžné technické praxi je často zapotřebí rozmístit konečný počet liniových nosných prvků (podélníků, resp. zakřivených nosníků) vyztužujících zatíženou rovinnou stěnu, resp. obecně definovanou válcovou plochu tak, aby tyto elementy byly namáhány stejnou intenzitou. Tato úloha (realisticky) předpokládá, že plošné konstrukce čelí spojitému rozložení zatížení po výšce svých površek a že zatěžující obrazec lze proto zobrazit prizmatem, resp. translační plochou, takže paralelně směrované nosné liniové prvky jsou v normálovém postavení k rovině spojitě definovaného zatížení. Příspěvek podává obecné řešení takto formulované úlohy, které je založeno na principech elementárního infinitezimálního počtu (vlastnostech součtových čar). Zvláštní pozornost je posléze věnována analytickému řešení pro „lichoběžné" a „trojúhelníkové" rozložení zatížení, která jsou v inženýrské praxi nejběžnější. Praktickým výsledkem článku bylo sestavení tabulek pro situování nosných liniových prvků v počtu do deseti kusů (včetně) při lineárním zatížení „trojúhelníkového" charakteru.

hydrostatický tlak; zemní tlak; opěrné zdi s horizontálními nosníky; zdi základů vystavené horizontálnímu zatížení; vertikální válcové (hlavně kruhové) skořepiny s tekutou nebo volně sypanou výztuží zesílenou horizontálními prvky (hlavně kruhy)

Contact Address:

Doc. Ing. Radimír Novotný, DrSc., Česká zemědělská univerzita v Praze, Fakulta lesnická a environmentální, katedra staveb, Kamýcká 1176, 165 21 Praha 6-Suchdol, Česká republika, e-mail: novotny@fle.czu.cz