SOFT EVALUATION OF ALTERNATIVES BY LANGUAGE OPERATORS*

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Article deals with expert approach to soft evaluation of alternatives. Qualified expert evaluates the benefits by means of natural language, using language operators. Natural language holds very detailed and important information that can be quantified using mathematical operations. The quantification procedure results in numerical cardinal weights assigned to proposed alternatives. Process of soft evaluation involves a) *provider* of evaluation, b) *alternatives,* c) *expert* who provides evaluation. Evaluation consists of four steps:

1. Identification of *alternatives A, B, C, ...,* which are subjects of evaluation. Alternatives consist of *objects* O_i and *statements* α_k. Alternatives can be either *oriented to object* or *oriented to statement.*

2. Choice of *language operators* $\$_j$, which must satisfy conditions of *ordinarity, selectivity, frequency, duality, transmissibility.* Language operators are proposed in form of a *menu* $\{\$_j\}$. There are two types of operators expert can use: a) strengthening language operators, and b) weakening language operators.

3. Creation of *evaluation chain* $\Im(A) = \{A| \$_1, \$_2, ..., \$_n\}$ for each proposed alternative *A*. Evaluation chain gives vague information expressed by words, but really it is an exact description of expert's opinion concerning proposed alternative.

4. Assignment of a sequence of measures μ_i , $\mu_i \in \langle 0, 1\rangle$ for each alternative. The first measure is assigned value $\mu_1 = 0.5$. Measures μ_i are modified by language operators $\{\$_j\}$ which expert puts in evaluation chain into a sequence. Modification is provided by means of *modification functions* MOD($\$_j$) which assign each operator a numerical value. Measure μ_m obtained in the final *m*-th step of algorithm can be considered as a *quantitative representation* of expert's evaluation of alternative. Generally, it is a cardinal weight assigned to alternative *A* on the interval $\langle 0, 1\rangle$. Modification function transforms language operator – which in fact is vague word – into quantitative numerical value. It modifies the initial measure of the alternative $\mu_1 = 0.5$ during step by step procedure $\mu_m = \mu_{m-1}[A| \text{MOD}(\$_m)]$. The process continues until all operators in the chain have been used for evaluation. Modification function for strengthening operators is concave on $\langle 0, 1\rangle$, for weakening operators is convex on $\langle 0, 1\rangle$. If MOD($) is concave modification function for a strengthening operator $, then MOD^{-1} ($) is convex modification function for the dual weakening operator, analogically. Two parametric functions were adopted within proposed methodology of soft evaluation, namely a) Zimmermann-Zysno function and b) Swarovsky function.As an example a procedure of evaluation of 8 alternatives in three steps evaluation is presented.

qualitative research; alternatives; objects, statements; evaluation chain; modification function; language operators; strengthening and weakening operators; soft/robust evaluation; fuzzy language operators; Zimmermann-Zysno function; Swarovsky function; quantification; cardinal weights; cardinal ordering

1. INTRODUCTION

Our daily life is filled with many technologies; highly sophisticated applications of ICT and/or on ICT based technologies change the manner of our lifestyle. Many citizens use PC, the Internet and mobile phones. It is not only private organizations, but also government, state administration, and other public institutions that use ICT in order to offer e-services, which are tailor-made to the needs of citizens.

Sophisticated use and implementation of ICT throughout the entire society may also bring problems: a heightened amount of digital divides in regards to the Internet and information systems. It seems that the world of "poor people", traditionally recognized as people who have little money, are today becoming monetarily wealthier. However, there is a new type of poverty emerging, and this is due to a lack of accessible ICT and information systems.

There exists a common agreement that ICT brings countless benefits to people. The government spends a vast amount of money to increase adult education with hopes of decreasing this digital divide, yet they have a vague objective: to improve the benefits of citizens, nobody doubts that ICT based technologies will result in anything but a positive result for both producers and citizens. However, there is one menacing question: What is the benefit for the citizens, and how can we measure it? In the case of producers, the benefit is clearly measurable. It is possible to compare and rate the amount of

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money invested with the amount of profit received. When dealing with citizens, the problem is not as clear. We can easily observe the amount of money invested, but are not easily able to quantify the benefits that citizens receive.

This problem is an emblematic one for *soft systems* and relevant soft decision making where benefit is immeasurable by means of traditional, robust measures. This article deals with expert approach to decision making in soft system. Many *knowledge oriented decision making problems* face to multi criteria decisions and thus often deal with soft evaluation of alternatives. Highly qualified experts evaluate the benefit of alternatives by means of natural language, using language operators rather than merely numerical forms. Natural language holds very detailed and important information that can later be quantified using mathematical operations. The result of this quantification procedure enables us to assign each object – primarily evaluated by means of words – a numerical cardinal weight.

2. MATERIAL AND METHODS

2.1. Algorithm of soft evaluation

The process of soft evaluation involves the cooperation of three factors:

1. *Provider* of evaluation (decision maker, manager, e.g. a mastermind who organizes the evaluation procedure).
2. *Alternative* (variant, proposition, programme, strategy, ...) which provider prepares and proposes for evaluation to an expert.
3. *Expert* who provides the evaluation (expertise) of alternative.

Step by step algorithm of evaluation consists of four following operations:

1. Identification of *alternatives A, B, C, ...,* which are subjects of evaluation. Alternatives consist of *objects* O_i and *statements* α_k.
2. Choice of appropriate *language operators* $\$_j$, which expert utilises for evaluation of proposed alternatives. The set of language operators $\{\$_j\}$ must satisfy following conditions:
   (a) *ordinarity*,
   (b) *selectivity*,
   (c) *frequency*,
   (d) *duality*,
   (e) *transmissibility*.
   Language operators are proposed to expert in the form of a *menu* $\{\$_j\}$. Mostly, the menu is presented as a list of language operators $\$_j$ ordered in a table.
3. Choice of an *expert* who evaluates proposed alternative. Expert provides evaluation by means of natural language – by words – using language operators presented in the menu $\{\$_j\}$.
4. Creation of *evaluation chain*

$$\Im(A) = \{A | \$_1, \$_2, \dots, \$_n\}$$

for the proposed alternative *A*. Evaluation chain is a sequence of language operators which are selected from the menu $\{\$_j\}$. Evaluation chain $\Im(A)$ of the alternative *A* is created in two steps:

a) *Initial step:*

Provider selects an appropriate language operator from the menu $\{\$_j\}$ and puts it on the first place into the evaluation chain; thus

$$\Im(A) = \{A | \$_1\}$$

in the first step.

The first evaluation chain of alternative *A* is assigned an initial measure

$$\mu_1 = \mu_1[\Im(A)] = \mu_1[A | \$_1] = 0{,}5,\ \mu_1 \in \langle 0, 1\rangle$$

Measure $\mu_1[A | \$_1] = 0.5$ is a numerical quantification of language operator $\$_1$. This initial value will be modified in next steps.

b) *Next steps:*

Process of evaluation is continued by expert. Expert evaluates the initial measure μ_1 which was assigned to alternative during the initial step. Expert is adding another language operators into the evaluation chain as long as she/he feels that the proposed alternative is well assessed. Finally, evaluation chain $\Im(A)$ of the alternative *A* consist of an ordered sequence of language operators, e.g. $\Im(A) = \{A | \$_1, \$_2, \dots, \$_n\}$.

Initial measure $\mu_1[\Im(A)] = 0.5$ is modified by other language operators $\$_2, \dots, \$_n$ which expert puts in evaluation chain into a sequence. Modification is provided by means of *modification functions* MOD($\$_j$) which assign each operator in the sequence a numerical value. It is a step by step procedure:

Let us suppose the initial measure $\mu_1[A | \$_1] = 0.5$ and expert put *m* other language operators $\$_1, \$_2, \dots, \$_m$ into the evaluation chain of the alternative *A*; then the initial measure $\mu_1[A | \$_1] = 0.5$ is transformed into modified values

$$\mu_2 = \mu_1[A | \$_1, \mathrm{MOD}(\$_2)],\ \mu_1[A | \$_1] = 0.5$$
$$\mu_3 = \mu_2[A | \$_1, \mathrm{MOD}(\$_2), \mathrm{MOD}(\$_3)],$$
$$\dots$$
$$\mu_i = \mu_{i-1}[A | \$_1, \mathrm{MOD}(\$_2), \mathrm{MOD}(\$_2), \dots, \mathrm{MOD}(\$_i)],$$
$$i = 2, 3, \dots, m.$$

Formally, evaluation procedure can be described as a sequence of quantifications of language operators which are ordered in evaluation chain:

$$\mu_1 = 0.5$$
$$\mu_2 = \mathrm{MOD}(\mu_1)$$
$$\mu_3 = \mathrm{MOD}(\mu_2)$$
$$\dots$$
$$\mu_m = \mathrm{MOD}(\mu_{m-1})$$

Value μ_m, obtained in the *m*-th step of algorithm, can be considered as a *quantitative representation* of expert's language evaluation of alternative *A*. Generally, it is a cardinal weight assigned to alternative *A* on interval $\langle 0, 1\rangle$.

2.2. Objects, statements, alternatives

Classical definition of a system as a *"set of elements and relations"* does not satisfy the requirements for a soft approach to evaluation. It is better to use the term *"class"* rather than *"set"*. When a system is considered to be a *"class of elements and relations"*, then each element has a *property* that symbolizes its membership to the class as well as to the system. The prerequisite that elements in systems have properties is a crucial one for soft evaluation.

Let us suppose we have a system in this new sense, and this is a soft system, e.g. some of its elements and/or relations are vague and not measurable by traditional robust measures. The basic unit we shall evaluate and/or assess is an *object (entity)*. The object can consist of:

a) one element or a collection of elements;
b) relation or collection of relations;
c) element and relation or a collection of elements and relations.

Objects can represent very different things and events. Objects can be robust, for example physical entities, which are well measurable by means of robust weights. Objects can be soft, for example philosophical or societal categories, which do not allow the use of traditional robust measures.

Here are examples of typical robust and soft objects:

Robust objects:

Car, men, temperature, expected value of future profit, probability of an event, estimation of future interest rate, … .

Soft objects:

Smell of a rose, smile of a child, benefit of farmers, welfare a citizen following from supermarkets, … .

We will assume that the properties of elements and relations are included in the properties of objects. Decision maker formulates the property of an object in the form of a *statement*. The set of chosen objects and the set of relevant statements make possible for the decision maker to formulate *alternatives*. Decision maker can formulate each alternative in two relevant forms, namely:

a) Alternative *oriented to object*: set of statements concerning *one object*.
b) Alternative *oriented to statement*: set of objects concerning *one statement*.

Example:

Capitals O_i denote objects, greek letters α_k denote statements, capitals *A, B, C*, … denote alternatives, respectively.

Let us suppose

O_1 = {leisure of worker},
O_2 = {leisure of clerk},
O_3 = {leisure of pensioner}

are objects, and that

α_1 = {is influencing family},
α_2 = {is influencing benefaction},
α_3 = {is influencing conviction},
α_4 = {is influencing outlook}

are statements. Object O_2 and statements α_1, α_2, α_3, α_4 make us possible to create many alternatives *oriented to the object*, e.g.:

A_{21} = {O_2 I α_1} = {leisure of clerk I is influencing family};
A_{22} = {O_2 I α_2} = {leisure of clerk I is influencing benefaction};
A_{23} = {O_2 I α_3} = {leisure of clerk I is influencing conviction};
A_{24} = {O_2 I α_4} = {leisure of clerk I is influencing outlook}.

These four alternatives could also be denoted only by one complete notation in the form: A_{2k} = {O_2 I α_1, α_2, α_3, α_4} = {leisure of clerk I is influencing family, is influencing benefaction, is influencing conviction, is influencing outlook}, k = 1, 2, 3, 4.

Here, expert balances and evaluates four statements from one fixed object point of view.

Similarly, we can compile alternatives oriented to the statement. For example, complete notation B_{4k} = {α_4 I O_1, O_2, O_3} = {is influencing outlook I leisure of worker, leisure of clerk, leisure of pensioner }, k = 1, 2, 3 presents three alternatives oriented to the statement α_4:

B_{41} = {α_4 I O_1} = {is influencing outlook I leisure of worker};
B_{42} = {α_4 I O_2} = {is influencing outlook I leisure of clerk};
B_{43} = {α_4 I O_3} = {is influencing outlook I leisure of pensioner}.

The expert balances and evaluates three objects from one statements point of view.

2.3. Choice of experts and planning of expertise

A high qualify expert can provide evaluations of both robust and soft objects using robust and soft measures; possible approaches of expert evaluation are evident in the Table 1.

A&B: Expert uses standard physical measures and gauges, evaluates by means of scales and other quantitative measures (often evaluating scales with five, seven or nine numbers are used. Also other techniques are used: Saaty methods, Fuller's method, etc.).

C&D: Expert evaluates object by means of language operators. Words like "many", "much", "a few", "undoubtedly", "unimportant", "trivial", "inconsiderable", "useful", "inevitable", and so forth, hold exact information and thus expert's own experience can be quantified by these common words.

Some special language operators are quantified in the formal fuzzy linguistics. Fuzzy language operators, for

Table 1. Evaluation of object by experts

		Object	
		robust	soft
Measures	robust	A	B
	soft	C	D

example words {very, highly, rather, more or less, roughly}, modify values of a given measure – so called "membership function" – by multiplying by a constant. This fuzzy approach does not give good results if there are more language operators in the evaluation chain.

2.4. Evaluation chain – menu of language operators for provider in the first step of evaluation

Sequence of language operators in the evaluation chain starts with language operator $\$_1$ which is put by provider in the first step of the evaluation algorithm. It is evident that in many cases language operators $\$_1$ = {good} or non$\$_1$ = {bad} will be used.

But, in the first step of evaluation provider can use also another language operators, see examples in Table 2.

Table 2. Example of menu of language operators for evaluation in the first step of evaluation

Language operators with constant MOD($\$_j$) = 0.5			
	positive		negative
$\$_1$		non$	
1	good	1	bad
2	excellent	2	not excellent
3	appropriate	3	not appropriate
4	accessible	4	not accessible
5	positively	5	negatively
6	substantially	6	not substantially
7	qualified	7	not qualified
8	healthy	8	diseased
9	pleasant	9	irritating
10	satisfactory	10	not satisfactory
	...		...

Note that language operators $\$_j$ and non$\$_j$ are in couples. Operator $\$_j$ has its dual form in operator non$\$_j$, and conversely.

2.5. Evaluation chain – menu of language operators for expert in next steps of evaluation

In next steps of evaluation expert expresses her/his agreement with the initial measure μ_1 = MOD($\$_1$) = 0,5. To do it expert ads into the evaluation chain other language operators. There are two types of operators expert can use:

a) strengthening language operators, and
b) weakening language operators.

Strengthening operators magnify initial measure μ_1 = 0.5 and push it to upper constraint of the interval $\langle 0.5; 1\rangle$.

Weakening operators make initial measure μ_1 = 0.5 lesser and push it to lower constraint of the interval $\langle 0; 0.5\rangle$.

The provider prepares menu of strengthening and weakening language operators and proposes it to expert. Example of menu is presented in Table 3.

Table 3. Example of menu of strengthening and weakening language operators

Language operators with different MOD($\$_j$)			
1	very	12	rather
2	highly	13	roughly
3	uniquely	14	nearly
4	irreproducible	15	rarely
5	uppermost	16	little
6	exceptionally	17	somewhat
7	enough	18	probably
8	surely	19	few
9	absolutely	20	more or less
10	totally		...
11	perfectly		...

2.6. Language operators

In each natural language there are frequent and commonly used words – language operators – by means of which expert expresses evaluation of proposed alternative. In the simplest case, expert uses robust two-value logic, e.g. makes decision that the initial evaluation of the alternative μ_1 = MOD($\$_1$) = 0.5 is true or is not true.

Soft evaluation of the alternative needs three-value logic (Havlíček, 1999a). Here, expert can use not only verdict "Yes" or "Not", but also evaluation running "between yes and not". The position "between Yes and Not" can be expressed by special words of natural language – by language operators. It is natural to depict values

{Yes, between Yes and Not, Not}

on the segment $\langle 0, 1\rangle$, e.g. on the continuum of real numbers, see Fig. 1.

Evaluation chain – containing a sequence of language operators – is created for each alternative. However, evaluation chain gives vague information expressed by words, but really it is an exact description of expert's opinion concerning proposed alternative. Thus, evaluation chain makes possible to give a precise picture of expert's opinion about alternative on the continuum "Between yes and not".

The menu of language operators must satisfy the following properties (Havlíček, 1999b):

1. Ordinarity

All language operators should be lined up in ascending (or descending) sequence according to their language strength.

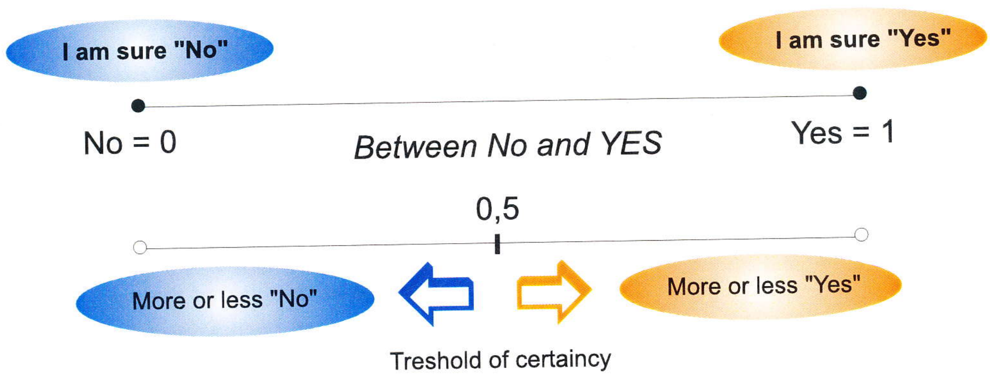

Fig. 1. Continuum of real numbers used for quantification of an expert's opinion

Example:

Operators {very, highly, uniquely, absolutely, totally, whole-heartedly} are lined up in the ascending mode; the operator "uniquely" can be considered as almost equal to the operator "absolutely", e.g. the sequence can be ordered this way:

{very} < {highly} < {uniquely} ≅
≅ {absolutely} < {totally} < {whole-heartedly}.

2. Selectivity

Each language operator should differ remarkably from other operators in the menu so that expert could select a "right" one and use it in order to strengthen or weaken the evaluation of the alternative on the continuum "Between Yes and Not".

3. Frequency

The list of operators should contain common words often used in daily and/or professional language. The use of *frequent vocabulary* in our modern language is recommended. Language operator "whole-heartedly" is an example of not appropriate language operator.

4. Duality

The initial value $\mu_1 = 0.5$, $\mu_1 \in \langle 0, 1 \rangle$ of each alternative is *increased* or *decreased* step by step by language operators ranked in the evaluation chain. Each language operator $\$_j$ has its *dual form* $\$_j^{-1}$. If language operator $\$_j$ is strengthening, then dual language operator $\$_j^{-1}$ is weakening, and conversely. Language operators thus can be presented *in couples* $\{\$_j, \$_j^{-1}\}$.

Dual symmetry between two language operators in a couple can be expressed by modification function as follows:

$$MOD(\$_j) = 1 - MOD^{-1}(\$_j)$$

MOD{strengthening operator} = 1 – MOD{weakening operator}.

Example:

The couple of mutually dual operators consists of a couple of different words, e.g.

MOD{much} = 1 – MOD{little},

or negation of the first operator can be used:

MOD{much} = 1 – MOD{not much}

5. Transmissibility

The choice of language operators should take into respect the requirements of a multilingual society, e.g. provider should involve into menu such operators that can be easily and unequivocally translated into other world languages. Of course, the expert will evaluate best in her/his native language.

2.7. Creation of evaluation chain

The evaluation chain of alternative *A*

$$\Im(A) = \{A|\ \$_1, \$_2, \ldots, \$_n\}$$

consists of several language operators, both strengthening and weakening. By testing the procedure in pilot applications, we found, that the best chains in practical applications contain at least three operators, e.g. $\Im(A) = \{A|\ \$_1, \$_2, \$_3\}$, where $\$_1$ was given by provider in the initial step and $\$_2$ and $\$_3$ were assigned by expert in two next steps of evaluation procedure.

Three levels of evaluation correspond well to the natural three superlatives in comparison of adjectives or adverbs, for example:

{good, better, the best}
{appropriate, more appropriate, the most appropriate}

Four or more operators in the chain resulted in higher frequency of weights near marginal points "Yes" and "No" in the continuum.

2.8. Modification function and process of quantification

Modification function $MOD(\$_j)$ is assigned to each language operator $\$_j$. Modification function transforms language operator – which in fact is vague word – into quantitative numerical value. It modifies the initial measure of the alternative $\mu_1 = 0.5$ during step by step procedure

$$\mu_1[A|\ \$_1] = 0{,}5$$
$$\mu_2 = \mu_1[A|\ \$_1, MOD(\$_2)]$$
$$\mu_3 = \mu_2[A|\ \$_1, MOD(\$_2), MOD(\$_3)],$$
$$\ldots$$
$$\mu_m = \mu_{m-1}[A|\ \$_1, MOD(\$_2), MOD(\$_3), \ldots, MOD(\$_m)].$$

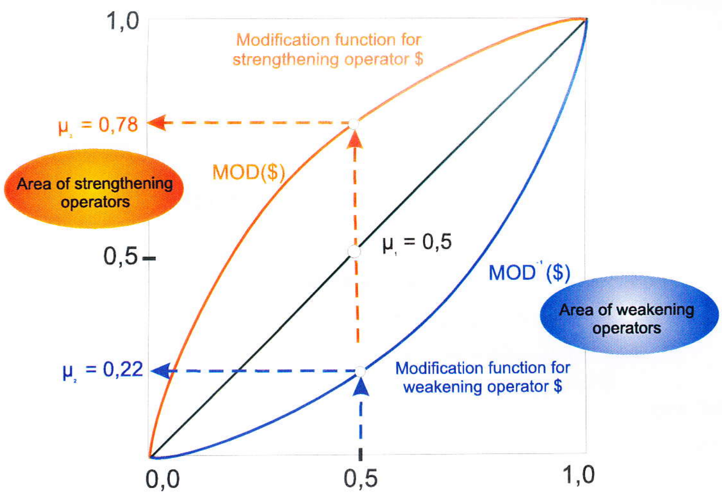

Fig. 2. Courses of modification function for a) strengthening operator, b) weakening operator

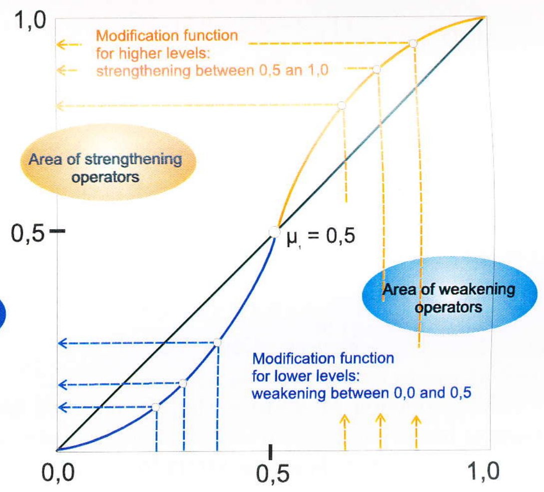

Fig. 3. Modification of Swarovsky function with one parameter μ

The process continues until all operators in the chain have been used for evaluation of alternative. Quantification of alternatives by means of modification function is thus performed in levels: each level of evaluation corresponds to one operator in the evaluating chain.

Fig. 2 depicts the transformation process – it shows how the modification function changes values μ_j into values μ_{j+1}. Strengthening modification function increased initial value $\mu_1 = 0.5$ to higher value $\mu_2 = 0.78$; analogically, weakening modification function decreased this value from $\mu_1 = 0.5$ to lower value $\mu_2 = 0.27$.

Modification function for strengthening operators is concave on $\langle 0, 1\rangle$, for weakening operators is convex on $\langle 0, 1\rangle$. If MOD($) is a concave modification function for a strengthening operator $, then MOD^{-1}($) is the convex modification function for the dual weakening operator, analogically.

2.9. Modification functions in fuzzy mathematics

Fuzzy mathematics defines modification functions for common language operators *very, highly, rather, rarely, slightly, more or less, roughly*:

Let $\mu_1 = \mu_1[\Im(A)] = \mu_1[A|\ \$_1] = 0.5$ is initial measure of alternative A, $\mu_1 \in \langle 0, 1\rangle$ and $\mu_2 = \mu_1[A|\ \mathrm{MOD}(\$_2)]$. Then

$\mu_2 = \mu_1[A|\ \mathrm{MOD}(\mathit{very})] = \mathrm{CON}(\mu_1) = \mu_1^2$,

$\mu_2 = \mu_1[A|\ \mathrm{MOD}(\mathit{highly})] = \mathrm{CON}[\mathrm{CON}(\mu_1)] = \mu_1^3$,

$\mu_2 = \mu_1[A|\ \mathrm{MOD}(\mathit{rather})] = [(2\mu^4 \wedge \mu^2) \vee (-2\mu_1^4 + 4\mu_1^2 - 1)]$,

$\mu_2 = \mu_1[A|\ \mathrm{MOD}(\mathit{rarely})] = \mu_1^{3.45}$,

$\mu_2 = \mu_1[A|\ \mathrm{MOD}(\mathit{slightly})] = \dfrac{2(\mu_1 \wedge (1-\mu_1^2))}{\sqrt{5}-1}$

$\mu_2 = \mu_1[A|\ \mathrm{MOD}(\mathit{more\ or\ less})] = \mathrm{DIL}(\mu_1) = 2\mu_1 - \mu_1^2$,

$\mu_2 = \mu_1[A|\ \mathrm{MOD}(\mathit{roughly})] = \mathrm{DIL}[\mathrm{DIL}(\mu_1)] =$
$= -\alpha^4 + \alpha^3 - 6\alpha_2 + 4\alpha.$

(Here, "CON", "DIL" are abbreviations of operations "contraction", "dilatation" – names of special functions in fuzzy mathematics; symbols "$\wedge$", resp. "$\vee$" denote "lesser from two values", "greater from two values", respectively. Examples:
$5 \wedge (-7) = -7$, $12 \vee 11 = 12$, $(\pi \wedge e) \vee (\sqrt{2} \wedge \sqrt[4]{1}) = e$).

2.10. Parametric modification functions

For other operators, as well as for standard fuzzy operators in higher levels of evaluation, new modification functions must be defined. During our pilot verification, these two artificial functions promise good results:

1) Zimmermann-Zysno function (Z i m m e r m a n n , 1985)

$$Z(x) = \frac{1}{2} + \frac{1}{d}\left[\frac{1}{1+e^{-a(x+b)}} - c\right]$$

with parameters a, b, c, d. This function is continuous and concave on interval $\langle 0, 1\rangle$ if parameter b = 0. The shape and slope of concavity on $\langle 0, 1\rangle$ can be improved by changing of parameters a, c, d. This type of function can be used for modification of measure μ by strengthening language operator $. Modification is provided by formula

$$\mathrm{MOD}(\mu) = \frac{1}{2} + \frac{1}{d}\left[\frac{1}{1+e^{-a\mu}} - c\right].$$

Relevant dual language operators can be quantified by relevant inverse Zimmermann-Zysno function; the function is continuous and monotonous on $\langle 0, 1\rangle$ and thus can be easily inverted. After calculations and using natural logarithm we obtain

$$Z^{-1}(x) = Z^{-1}(x) = -\frac{1}{a}\left[\ln\frac{1-d(x-0.5)-c}{d(x-0.5)+c}\right] - b.$$

Inverse Zimmermann-Zysno function $Z^{-1}(x)$ is continuous and convex on interval $\langle 0, 1\rangle$ for b = 0 and its shape and slope can be improved by changing of parameters a, c, d. This function can be used for modification of measure μ by weakening operator $; modification can be provided by formula

$$\mathrm{MOD}^{-1}(\mu) = -\frac{1}{a}\left[\ln\frac{1-d(\mu-0.5)-c}{d(\mu-0.5)+c}\right].$$

On the Fig. 2 the shapes of functions MOD(μ) and $\mathrm{MOD}^{-1}(\mu)$ are depicted for parameters a = 3, b = 0, c = 0, d = 0.4525:

$$\mathrm{MOD}(\mu) = \frac{1}{2} + \frac{1}{0.4525}\left[\frac{1}{1+e^{-3\mu}}\right].$$

$$\mathrm{MOD}^{-1}(\mu) = -\frac{1}{3}\left[\ln\frac{1-0.4525(\mu-0.5)}{0.4525(\mu-0.5)}\right].$$

Here, function MOD(μ) is slightly selective and was used to modify measure μ = 0.5 by strengthening operator {$} = {surely}. Function $\mathrm{MOD}^{-1}(\mu)$ modify the measure μ = 0,5 by operator {$} = {not surely} = {probably}.

2) Swarovsky function

$$F(x) = \frac{1}{2} + \frac{1}{2}\sin\left\{\frac{\pi}{b-a}\left(x - \frac{a+b}{2}\right)\right\}$$

was adopted to the shape

$$\mathrm{MOD}(\mu) = \frac{1}{2} + \frac{1}{2}\lambda\sin\left\{\pi\left(\mu - \frac{1}{2}\right)\right\}$$

with one parameter λ. The function has a sigmoid shape and has one point of inflection in $\left\langle\frac{1}{2}, \frac{1}{2}\right\rangle$.

Swarovsky function can be used:

a) in cases where the initial value μ_1 of an alternative is not the midpoint of ⟨0, 1⟩, e.g. $\mu_1 \neq 0.5$, $\mu_1 \in \langle 0, 1\rangle$, and

b) in higher levels of the quantification process, if weights $\mu_j > 0.5$ or $\mu_j < 0.5$ (where symbols ">" , resp. "<" denote "markedly greater than", resp. "markedly lower than").

The shape of Swarovski function is depicted on Fig. 3.

3. RESULTS – EXAMPLE

Decision maker proposed 8 alternatives {A, B, C, D, E, F, G, H}. Expert was asked to evaluate all 8 alternatives.

Evaluation was realised in three steps. In the first step of evaluation decision maker proposed each alternative language operator $\$_1$ = {satisfactory} with MOD($\$_1$) = μ_1 = 0.5. Then expert continues the process and put two other operators into evaluation chains of alternatives. Thus, we obtained following evaluation chains:

$\Im(A)$ = [*A*| {satisfactory}, {rather}, {seldom}],
$\Im(B)$ = [*B*| {satisfactory}, {very}, {seldom}],
$\Im(C)$ = [*C*| {satisfactory}, {seldom}, {seldom }],
$\Im(D)$ = [*D*| {satisfactory}, {seldom}, {undoubtedly}],
$\Im(E)$ = [*E*| {satisfactory}, {rather}, {undoubtedly}],
$\Im(F)$ = [*F*| {satisfactory}, {very},{undoubtedly}],
$\Im(G)$ = [*G*| {satisfactory}, {highly}, {seldom}],
$\Im(H)$ = [*H*| {satisfactory}, {highly}, {undoubtedly}].

Modification of the initial value μ_1 = 0.5 in the second level of evaluation was provided by fuzzy modification functions (see paragraph 2.9). Results of evaluation in the second level are presented in Table 4.

Table 4. Quantification of alternatives in second level of evaluation

2nd level of evaluation				
Alternative	$\$_j$	μ_1	μ_2	Order
G	highly	0.5	0.875	1
H	highly	0.5	0.875	1
B	very	0.5	0.750	2
F	very	0.5	0.750	2
A	rather	0.5	0.125	3
E	rather	0.5	0.125	3
C	seldom	0.5	0.091	4
D	seldom	0.5	0.091	4

Modification of measures μ_2 in the third level of evaluation was provided by modified Zimmermann-Zysno function. Results of modification and final ordering of alternatives are presented in Table 5.

Table 5. Quantification of alternatives in third level of evaluation

3rd level of evaluation					
Alternative	$\$_j$	μ_1	μ_2	μ_3	Order
G	highly, seldom	0.5	0.875	0.717	3
H	highly, undoubtedly	0.5	0.875	0.955	1
B	very, seldom	0.5	0.750	0.551	4
F	very, undoubtedly	0.5	0.750	0.894	2
A	rather, seldom	0.5	0.125	0.075	7
E	rather, undoubtedly	0.5	0.125	0.204	5
C	seldom, seldom	0.5	0.091	0.055	8
D	seldom, undoubtedly	0.5	0.091	0.148	6

Measures {μ_3} are cardinal weights assigned to alternatives. Ordinary ordering of alternatives is {H, F, G, B, E, D, A, C}.

The weight of alternative C, which is derived from chain {satisfactory, seldom, seldom}, is an example of use of two same operators.

4. CONCLUSION

The methodology represents a very general approach to quantification of information presented in natural language. Concrete use and application of this method depend on concrete problems. The role of decision maker (manager) is crucial: he must prepare alternatives, menus of appropriate language operators for experts, provide choice of experts, adjust modification functions and process the data.

The method can be used in qualitative research. In qualitative research, many quantitative approaches terminate in an ordinary sequence of alternatives, e.g. in defining their order in a linear fashion. Qualitative researchers rarely use complicated mathematical methods. Statistics and probability often are used, but they are - in the context of this article – considered to be robust methods. The presented method makes possible to assign cardinal weights to each alternative and thus give more detailed information and superior ways to measure quality.

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Měkké hodnocení alternativ pomocí jazykových operátorů.

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Článek se zabývá hodnocením alternativ pomocí expertů. Kvalifikovaný expert hodnotí kvalitu alternativy slovy přirozeného jazyka, tzv. jazykovým operátorem. V přirozeném jazyce expert vyjadřuje přesnou informaci, která se může kvantifikovat matematickými postupy. Proces kvantifikace končí přiřazením kardinálních vah každé z hodnocených alternativ.

Procesu evaluace se účastní a) *organizátor evaluace* (manažer), b) *alternativy* (varianty, návrhy, projekty, programy, strategie, apod.), c) *expert*, který alternativy hodnotí.

Evaluace se provádí ve čtyřech krocích:

1. Identifikace a příprava *alternativ A, B, C, ...*, které budou expertem hodnoceny. Alternativa se skládá ze dvou částí: *objektu* O_i *a tvrzení* α_k. Alternativy se mohou expertovi předkládat ve dvou variantách jako alternativy *orientované na objekt*, nebo alternativy *orientované na tvrzení*.

2. Výběr jazykových operátorů $\$_j$ které musí splňovat podmínky *ordinarity, selektivity, frekvence, duality, přenosu*. Jazykové operátory se předkládají expertovi k použití ve formě *menu* $\{\$_j\}$.

3. Tvorba evaluačního řetězce $\Im(A) = \{A| \$_1, \$_2, \dots, \$_n\}$ pro každou alternativu *A*. Evaluační řetězec podává o každé variantě sice vágní informaci vyjádřenou slovy, ve skutečnosti ale tato informace obsahuje zkušenost experta a přesně vyjadřuje jeho mínění o hodnocené alternativě.

4. Každé alternativě se přiřazuje míra její kvality μ_i , $\mu_i \in \langle 0, 1\rangle$.

Míry μ_i se modifikují (zpřesňují) pomocí jazykových operátorů $\{\$_j\}$, které expert postupně přidává do evaluačního řetězce. Modifikace se provádí pomocí speciálních *modifikačních funkcí* MOD($\$_j$), které každému jazykovému operátoru přiřazují numerickou hodnotu. Míru μ_m, získanou na konci m-tého kroku algoritmu, můžeme považovat za kvantitativní reprezentaci expertova hodnocení. Obecně lze tuto hodnotu považovat za kardinální váhu alternativy *A* na intervalu hodnocení $\langle 0, 1\rangle$.

Jazykové operátory ve fuzzy matematice, např. výrazy velmi, značně, dosti, víceméně, zhruba, modifikují tzv. funkci příslušnosti do fuzzy množiny násobením konstantními hodnotami. Tento postup nezaručuje dobré výsledky v případě, kdy je v evaluačním řetězci větší počet různých jazykových operátorů.

Expert může při hodnocení alternativy použít dva typy jazykových operátorů: a) *operátory zesilující*, které zvyšují hodnotu míry μ a posouvají ji k horní hranici intervalu $\langle 0,5; 1\rangle$, b) *operátory zeslabující*, které zmenšují hodnotu míry μ a posouvají ji směrem k dolní hranici intervalu $\langle 0; 0,5)$.

Každý jazykový operátor $\$_j$ má svoji *duální formu* $\$_j^{-1}$. Jestliže je operátor $\$_j$ zesilující, potom je operátor $\$_j^{-1}$ = non$\$_j$, a naopak. Jazykové operátory se proto předkládají v *párech* $\{\$_j, \$_j^{1}\}$. Duální symetrický vztah mezi dvěma jazykovými operátory lze vyjádřit pomocí modifikačních funkcí takto: MOD($\$_j$) = 1 - MOD^{-1} ($\$_j$).

Evaluační řetězec $\Im(A) = \{A| \$_1, \$_2, \dots, \$_n\}$ se skládá z několika operátorů, které mohou být zesilující i zeslabující. V pilotním ověřování tohoto postupu jsme zjistili, že nejlepší výsledky dávají evaluační řetězce, které obsahují tři jazykové operátory, tj. řetězce $\Im(A) = \{A| \$_1, \$_2, \$_3\}$. Manažer, který organizuje evaluaci, vybere první jazykový operátor $\$_1$ a zadá mu výchozí míru $\mu_1 = 0{,}5$. Zbývající dva operátory $\$_2$ a $\$_3$ vloží potom do řetězce expert ve dvou následujících krocích evaluace.

Modifikační funkce MOD($\$_j$) se připravuje pro každý jazykový operátoru z menu $\{\$_j\}$. Modifikační funkce transformuje jazykový operátor – který je v podstatě vágní slovo – na kvantitativní numerickou hodnotu z intervalu $\langle 0, 1\rangle$. Modifikační funkce v průběhu algoritmu postupně upravuje výchozí míru $\mu_1 = 0{,}5$ na hodnoty $\mu_m = \mu_{m-1}[A|$

$MOD(\$_m)]$, m = 1, 2, … . Proces končí, když se zpracují všechny operátory zařazené do posloupnosti v evaluačním řetězci.

Modifikační funkce pro zesilující operátory je konkávní na ⟨0, 1⟩, pro zeslabující operátory je konvexní na ⟨0, 1⟩. Jestliže je MOD($) konkávní pro zesilující operátor $, potom je inverzní funkce $MOD^{-1}(\$)$ konvexní pro duální zeslabující operátor, a naopak.

V rámci výzkumu byly pro potřeby evaluace vybrány a upraveny dvě funkce, jejichž průběh lze vhodně upravovat pomocí parametrů a tak je přizpůsobit pro vyjádření „síly" jazykového operátoru. Jsou to funkce:

a) Zimmermann-Zysnova funkce Z(x), kterou lze použít pro kvantifikaci zesilujících operátorů. Odpovídající inverzní funkce $Z^{-1}(x)$ naopak kvantifikuje operátory zeslabující. Průběh funkce je možno upravovat pomocí tří parametrů.

b) Funkce Swarovského s parametrem λ je vhodná pro kvantifikaci jazykových operátorů v případech a) kdy počáteční míra $\mu_1 \neq 0,5$, $\mu_1 \in \langle 0, 1\rangle$, a b) když v hodnotícím řetězci je větší počet různých jazykových operátorů a postup hodnocení vyžaduje větší selektivitu hodnotících funkcí.

Jako ilustrační příklad se uvádí hodnocení osmi alternativ s hodnotícími řetězci se třemi jazykovými operátory.

kvalitativní výzkum; alternativy; objekty; tvrzení; evaluační řetězec; modifikační funkce; jazykové operátory; zesilující a zeslabující operátory; měkká/robustní evaluace; fuzzy jazykové operátory; Zimmermann-Zysnova funkce; funkce Swarovského; kvantifikace; kardinální váhy; kardinální uspořádání

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