MATHEMATICAL MODELS FOR ELEMENTARY KNOWLEDGE REPRESENTATION*

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Mathematical models are being used during system analysis of real object problems. They use mathematical apparatus for description of abstract system elements and relationships. Knowledge and mathematical model have some common properties. They have strong relationship to problems situation solution. Mathematical model is a tool for finding a solution, knowledge is a product of experience with successful implementation of that solution. This article deals with mathematical models as knowledge representation. Defines a term of "elementary knowledge" as a specific type of knowledge unit and shows its possible representation in concrete mathematical model.

knowledge; knowledge as an object; knowledge unit; problem situation; complex problem; elementary problem; elementary knowledge; mathematical model

INTRODUCTION

Knowledge management and mathematical modeling

Two main strategies for knowledge management have been employed by early adopters of the principle: "product" and "process" approaches.

Following Hansen et al. (1999) and Mentzas et al. (2001), the "product" approach implies that knowledge is a thing that can be located and manipulated as an independent object. Adopting the "knowledge as a product" approach means treating knowledge as an entity rather separate from the people who create and use it. This approach mainly focuses on products and artefacts containing and representing knowledge; usually, this means managing documents, their creation, storage, and reuse in computer-based corporate memories.

The "process" approach puts emphasis on ways to promote, motivate, encourage, nurture or guide the process of knowing, and abolishes the idea of trying to capture and distribute knowledge. This view mainly understands knowledge management as a social communication process, which can be improved by collaboration and cooperation support tools. In this approach, knowledge is closely tied to the person who developed it and is shared mainly through person-to-person contacts.

Mathematical models are being used during system analysis of real object problems. They use mathematical apparatus for description of abstract system elements and relationships.

From the other point of view the mathematical model can be understood as a special type of knowledge representation, very suitable for knowledge map construction. Davenport and Prusak (2000) note that developing knowledge map involves locating important knowledge in the organization and then publishing some sort of list or picture that shows where to find it. Knowledge maps typically point to people as well as to document and databases.

This article is appointed to the knowledge as an object and deals with mathematical models as knowledge representation. Defines a term of "elementary knowledge" as a specific type of knowledge unit and shows its possible representation in general mathematical model.

MATERIAL AND METHODS

Data, information, knowledge

Let us go out from generally known definitions of data, information and knowledge (Ackoff, 1989)

Data

Data is raw. It simply exists and has no significance beyond its existence (in and of itself). It can exist in any form, without relation to usability. It does not have meaning of itself. In computer parlance, a spreadsheet generally starts out by holding data.

Data are distinct pieces of information usually formatted in a special way. Data can exist in a variety of forms: as numbers or text on pieces of paper, as bits and bytes stored in electronic memory, or as facts stored in a person's mind.

The term data is often used to distinguish binary machine-readable information from textual human-readable information. For example, some applications make a dis-

* The paper is supported by the grant project of Ministry of Education Czech Republic No. MSM6046070904 – "Information and Knowledge Support of Strategic Management".

tinction between data files (files that contain binary data) and text files (files that contain ASCII data).

Information

Information is data that has been given meaning by way of relational connection (Ackoff, 1989). This "meaning" can be useful, but does not have to be. In computer parlance, a relational database makes information from the data stored within it. Information is a flow of messages. The patterns and relationship in the data is pointed out and discussed. The data is made informative and must be put into a context and linked like data.

Knowledge

There are many attempts to define what would be the exact semantics for the term "knowledge", what would be the difference between "knowledge" and "information", and what would be the elementary pieces of knowledge, sometimes called knowledge units (Brožová, Klimešová, 2006).

Knowledge is defined as an ordered set of information in space and time about important notions, data, facts, axioms, laws, and inference rules related to a specified field of human experience, embedded in a given thought-framework (Roska, 2003). In what follows, information without a thought-framework will not be qualified as knowledge.

Relationships between data, information and knowledge are often described in a form of hierarchical structure (pyramid), see Fig. 1.

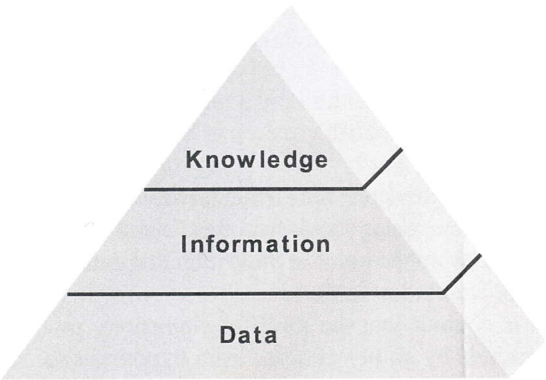


Fig. 1. Data, information and knowledge

Data is a raw digital material or the "artifacts, which exist as a vehicle for conveying information". Information is interpreted "within a context set by a priori knowledge and the current environment". Knowledge assigns a purpose and/or action to information. Knowledge is based on "information integrated in a fashion which allows it to be used in further interpretation and analysis of data".

Knowledge representation

Generally, knowledge can be represented by several ways. Usually, production rules, semantic networks and frames are used for this purpose.

- Production rules

One of the most popular approaches to knowledge representation is to use production rules, sometimes called IF-THEN rules. They can take various forms, e.g.

- IF condition THEN action
- IF premise THEN conclusion
- IF proposition p1 and proposition p2 are true THEN proposition p3 is true

Some of the benefits of IF-THEN rules are that they are modular, each defining a relatively small and, at least in principle, independent piece of knowledge. New rules may be added and old ones deleted usually independently of other rules.

- Semantic networks

These are large networks of ideas containing a multitude of concepts. The links show the relations between them. They are normally generated with the aid of computer software. Pairs of related concepts are joined by bi-directional links. Like concept maps, semantic networks capture concept microstructure and topic macrostructure. Semantic networks are said to be easy to understand by humans and can also be used in automated processing systems. They are often used as a learning tool within the educational field (Gordon, 2000).

- Frames

A frame is a data structure that can be used for knowledge representation. Roughly similar to the object-oriented paradigm, they represent classes (called frames) with certain properties called attributes or slots whereas they do not have methods. Frames are thus a machine-usable formalization of concepts or scheme.

Otherwise, decision tables, decision trees or scenarios can be used for knowledge representation. Their description is out of range of this article.

Mathematical modeling

Mathematical modeling is a process of using the mathematical language to describe the behavior of a system. Mathematical models are used particularly in the sciences such biology, electrical engineering, physics but also in other fields such as economics, sociology and political science.

Often when engineers analyze a system to be controlled or optimized, they use a mathematical model. In analysis, engineers can build a descriptive model of the system as a hypothesis of how the system could work, or try to estimate how an unforeseeable event could affect the system. Similarly, in control of a system, engineers can try out different control approaches in simulations (Šubrt, Dömeová, 2002).

A mathematical model usually describes a system by a set of variables and a set of equations that establish relationships between the variables. The values of the variables can be practically anything; real or integer numbers, boolean values or strings, for example. The variables represent some properties of the system, for example, measured system outputs often in the form of signals, timing data, counters, event occurrence (yes/no). The actual model is the set of functions that describe the relations between the different variables.

According to B e r k a (2002), the most important properties of mathematical models are as follows:

1. Generality – It is possible to use one mathematical model for many object properties and intra-relationships description.
2. Shortness and accuracy – There are a lot of implicit knowledge about object in mathematical models, which can be converted to explicit using exact mathematical tools.
3. Simple verification of hypotheses – Hypotheses can be formulated exactly and proved by mathematical methods.

These properties of mathematical models will be used for knowledge representing, description and mapping.

RESULTS

Knowledge unit, elementary knowledge

The basic structural element of knowledge is knowledge unit. Z a c k (1999) defines the knowledge unit as an atomic packet of knowledge content that can be labeled, indexed, stored, retrieved and manipulated. The format, size and content of knowledge units may vary depending on the type of explicit knowledge being stored and the context of their use.

There are many similar definitions in literature. Unfortunately, most of them are very general and not suitable for connection to the structure of mathematical model. Let us define the term "elementary knowledge" as a specific type of knowledge unit. Inspired by S l o u k o v á (2004), we define the elementary knowledge as follows:

Elementary knowledge EK is a basic unit of knowledge. It is one predictive hypothesis, which has following form: Let's have a situation X and a problem Y. With regards to objective(s) Z we can expect consequences Q. Formally,

$$EK = \{X, Y, Z, Q\}.$$

Elementary knowledge is shown in the Fig. 2.

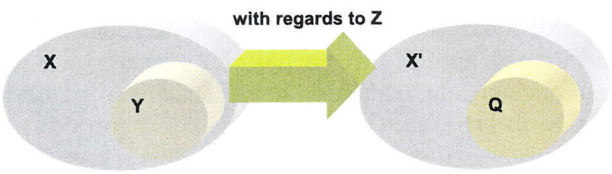


Fig. 2. Elementary knowledge

A problem Y is a part of problem situation X. Due the predicate hypothesis in the form "IF condition THEN result" the knowledge is given and the consequence influencing the problem situation X is determined. The problem situation X is moving to the state X', better from the point of view Z.

There are two key presumptions of definition given above:

1) Connection to a problem situation

There is no knowledge (neither elementary knowledge) without connection to some problem situation. Each of knowledge is born as a result of **successful** solution of some problem.

2) Hierarchical level of elementary

Elementary knowledge was born as an elementary problem solution. Elementary problem is a problem or part of some complex problem, which is not useful to split to more simple sub-problems. It is in concordance with Zack's definition of knowledge unit ("...atomic packet of knowledge content...").

Especially user of knowledge determines criteria of elementary level setting. They depend on his ability to follow the instructions included in the elementary knowledge.

Example

We can show the example of elementary knowledge definition. Let's have the elementary problem: when to leave the house to catch the bus, which leaves at 7:45. The elementary knowledge connected to this problem can be defined as follows:

"IF I leave the house till 7:30, (THEN) I will catch the bus at 7:45."

Following the statement EK = {X, Y, Z, Q}, we can determine, that

X = "my position"
Y = "when to leave the house"
Z = "to catch the bus"
Q = "going by bus"

It is possible to test the elementary level of the knowledge. Decomposing both hypothesis and consequence statement from the point of view included data and information, we obtain Table 1.

It is obvious that the level for elementary problem is determined by ability of actor from hypothesis to do the action from hypothesis. In the problem above, if I know, what to do to leave the house till 7:30, the problem of bus catching is elementary problem for me and the knowledge for solving that problem can be considered to be elementary knowledge.

The knowledge does not include anything about my knowledge the way from the house to the station, stochastic character of walking time to the station and many other factors, which influence the achievement of the goal "catching the bus". It is correct because of way of definition "Y".

Table 1. Elementary knowledge decomposition

Elementary knowledge
IF I leave the house till 7:30, **THEN** I will catch the bus at 7:45.

Hypothesis information
I leave the house till 7:30.

Consequence information
I catch the bus at 7:45.

Hypothesis data			
Deadline	Place	Action	Actor
...	...	...	...
7:20	street	**leave**	**me**
7:25	**house**	wake up	my father
7:30	square	eat	my mother
7:35	station	wash	my sister
...	...	...	...

Consequence data			
Departure time	Vehicle	Action	Actor
...	...	...	...
7:15	**bus**	miss	**me**
7:30	car	**catch**	my father
7:45	tram	ignore	my mother
8:00	metro	use	my sister
	...	...	...

DISCUSSION

Mathematical model for knowledge representation

In general, mathematical model is used for reality or its simplification (abstract system) representation. A person, who is skilled in mathematical modeling, can exploit that form of representation to analyze what happened in past, what is now in presence and what will happen in the future.

Mathematical model is very efficient form of knowledge representation. The mathematical model is
- highly structured, created following systems methodology, composed by exact defined elements,
- highly abstracted, general, prepared for implementation in similar cases,

and, when the model is well interpreted, it is possible to use it for other knowledge storing or inference some new ones. This is the most important difference between a mathematical model and a story as two forms of knowledge representations.

There is other aspect for support the idea of using mathematical model as a knowledge representation. It is strong similarity of life-cycle process of both tools:

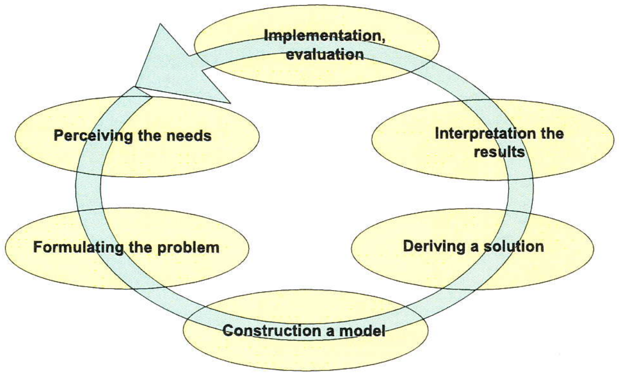


Fig. 3. Life cycle of mathematical model for knowledge representation

1) Perceiving the needs
2) Formulating the problem
3) Construction a model
4) Deriving a solution
5) Interpretation the results
6) Implementation, evaluation

The process of that life cycle is shown in Fig. 3.

The elementary knowledge EK = {X, Y, Z, Q} is a relationship between the hypothesis "Y" and the consequence "Q". It is some kind of assignment, which is valid in the range of problem situation "X" with regards to objective "Z".

Generally, such assignment of entities can be modeled by a function. The first part of elementary knowledge (hypothesis "Y") can be scaled on axis x (independent variable), the other part (consequence "Q") on axis y. Quality of information scaled on the axis (nominal, ordinal, cardinal) does not matter for now. That access to elementary knowledge modeling can be shown on illustrative example.

Example

Let's have a part of fiction production system described as follows:

"A manufacturing company produces product P and ... . All products must be processed through a machine. The machine has 12 hours of available capacity. Each unit of product P requires 3 hours of time on the machine, ... The profit is $4 per unit of product P, ... The company can sell as many units of all products as it can produce. The objective of the company is to maximize profits... ."

Following elementary knowledge can be drawn from the description in Table 2.

Analyzing the elementary knowledge structure, we obtain Tables 3 and 4.

In this case, any elementary knowledge is represented by direct proportion function. Functional dependencies between hypotheses and consequences included in elementary knowledge can be described as it is shown in Fig. 4.

Table 2. Elementary knowledge components

Elementary knowledge	X	Y	Z	Q
EK1	production	Machine time for producing P spent	Produce 1 unit of product P	1 unit of product P finished
EK2	profit	Product P sold	Realize the profit for 1 unit of product P	1 unit of product P realized

Table 3. Elementary knowledge for production problem (production)

Elementary knowledge 1				
IF a unit of product P is produced, **THEN** 3 hours of the machine 12 hours capacity will be used.				
Hypothesis information			**Consequence information**	
1 unit of product P is produced.			3 hours of the machine 12 hours capacity are used.	
Hypothesis data			**Consequence data**	
Amount	Product	Action	Machine capacity	Action
1 unit	product P	produced	3 hours from 12	used

Table 4. Elementary knowledge for production problem (profit)

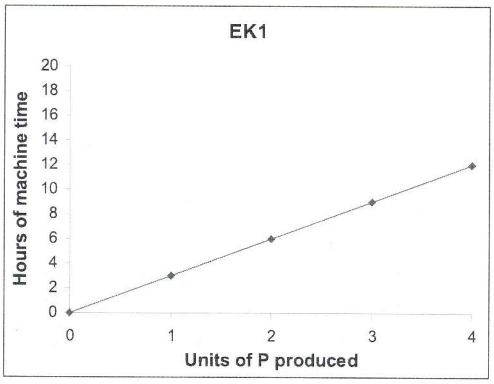
Elementary knowledge 2				
IF a unit of product P is sold, **THEN** 4$ of profit will be realized.				
Hypothesis information			**Consequence information**	
1 unit of product P is sold.			4$ of profit are realized.	
Hypothesis data			**Consequence data**	
Amount	Product	Action	Profit	Action
1 unit	product P	sold	4$	realized

Now, it is simple to use that elementary knowledge as a part of mathematical model. Product P could be represented as one of this activity, functional dependencies derived from elementary knowledge as constraints and/or objective function.

CONCLUSIONS

Mathematical models are often used during systems analysis procedure for solving many complex problems. Their potential as some specific type of knowledge representation is demonstrated in the article. Mathematical models are very powerful tool for knowledge representing, especially for their generality, accuracy and structure. They respect hierarchy of complex problem, which requires knowledge in the form of knowledge map, and also allows solving partial (elementary) problem and store/save/inference knowledge units.

The definition of elementary knowledge as a specific type of knowledge representation was suggested in the article. Elementary knowledge has a standardized form of implication hypothesis – consequence, which is valid in the environment of the defined problem situation,



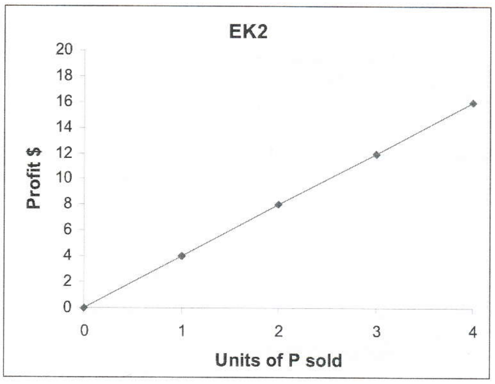


Fig. 4. Functions for elementary knowledge representation

which is evaluated from the point of view selected objective. Even objective, connection to the problem solving and dynamics, these factors are making knowledge and make sense of knowledge mapping and modeling.

REFERENCES

ACKOFF, R. L.: From Data to Wisdom. J. Appl. Systems Analysis, *16*, 1989: 3–9.

BERKA, M.: Operační výzkum. SF VUT Brno, 2002

BROŽOVÁ, H., KLIMEŠOVÁ, D.: Prostor pro komunikaci znalostí. In: Proc. Conf. ERIE, Praha, 2006.

DAVENPORT, T. H. – PRUSAK, L.: Working Knowledge. Harvard Business School Press 2000.

GORDON, J. L.: Creating Knowledge Maps by Exploiting Dependent Relationships. Knowledge Based Systems, *13*, 2000: 71–79.

HANSEN, M. T. – NOHRIA, N. – TIERNEY, T.: What's Your Strategy for Managing Knowledge? Harvard Business Review, 1999: 107–116.

MENTZAS, G. – APOSTOLOU, D. – ABECKER, A.: Managing Knowledge as a Strategic Resource For Electronic Government. In: Workshop "Electronic Government and Knowledge Management", Siena, 2001.

ROSKA, T.: Information based society and knowledge. World Science Forum, Budapest, 2003.

SLOUKOVÁ, D.: Znalost z pohledu sémiotiky a hermeneutiky, a orientace znalostního managementu. E-Logos 2004.

ŠUBRT, T. – DÖMEOVÁ, L.: Multi Criteria Multilevel Transshipment Problem and Its Software Support. In Advances in Soft Computing – Multi Objective and Goal Programming, Advances in Soft Computing – Multi Objective and Goal Programming. Springer 2002: 377–385.

ZACK, M. H.: Managing Codified Knowledge. Sloan Management Review, *40*, 1999 (4): 45–58.

Received for publication on June 12, 2006
Accepted for publication on August 21, 2006

HOUŠKA, M. – BERÁNKOVÁ, M. (Česká zemědělská univerzita, Fakulta provozně ekonomická, katedra operační a systémové analýzy, Praha, Česká republika):

Reprezentace elementárních znalostí pomocí matematického modelu.

Scientia Agric. Bohem., *37*, 2006, Special Issue: 32–37.

Na znalost je možné nahlížet ze dvou úhlů pohledu: jako na produkt (objekt) a jako na proces. V prvním případě je znalost chápána jako entita, kterou je možno vymezit a zpracovávat. Ve druhém případě je hlavní důraz kladen na podporu, motivaci a vedení poznávacího procesu jako zdroje získávání znalostí.

Matematické modely se používají v procesu systémového řešení reálných problémů jako nástroj pro formalizaci rozhodovacího prostoru s cílem nalézt řešení, které z hlediska daného cíle vyřeší problémovou situaci nebo ji alespoň zlepší. Matematické modely jsou vždy konstruovány v kontextu problémové situace a volba konkrétního modelu a úrovně podrobnosti modelového zobrazení je podřízena cíli analýzy.

Znalost vzniká jako výsledek úspěšného řešení nějakého problému. Problému, který někdo identifikoval, vyřešil, řešení implementoval a ověřil jeho úspěšnost. Na povaze problému a jeho řešení závisí to, zda získaná znalost měla formu explicitní, možnou ke sdílení, nebo tacitní, nesdělitelnou.

V návaznosti na problémovou situaci vzniká zároveň vazba mezi znalostí a matematickým modelem. Jestliže matematický model slouží k popisu a řešení problémové situace a znalost je výsledkem řešení problému s ověřenou úspěšností, je možné považovat matematický model za specifickou formu reprezentace výsledné znalosti.

Článek se zabývá znalostí jako objektem. Jeho cílem je ukázat, jak je možné využít matematický model pro reprezentaci znalostí. Pro tento účel je definován pojem „elementární znalost" jako specifický, dobře strukturovaný typ znalostní jednotky jako atomické jednotky znalosti. Atomické v tom smyslu, že ji z hlediska jejího uživatele není možné a/nebo účelné dále dělit. V článku je analyzována struktura elementární znalosti ve vztahu k datům a informacím, které jsou v ní obsaženy. Na konkrétním příkladu matematického modelu lineárního programování je ukázáno, jak je možné elementární znalosti identifikovat a zobrazovat pomocí prostředků tohoto matematického modelu.

znalost; znalost jako objekt; znalostní jednotka; problémová situace; komplexní problém; elementární problém; elementární znalost; matematický model

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