COMPARISON OF NORWAY SPRUCE (*PICEA ABIES* [L.] KARST.) STEM SHAPE IN TWO DIFFERENT LOCALITIES

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This study deals with the stem shape evaluation of Norway spruce in two different localities. Thirty two stems from the School Training Forest Kostelec nad Černými lesy and 32 stems from the forest district Lenora lying in the Protected Landscape Area the Šumava Mts. were investigated. These 64 stems became the base for the reduced analysis. Both localities were represented by 64 stems: (sets Lenora overbarks (32 stems) and Doubravčice overbarks (32 stems)) and another 256 stems without bark (Lenora underbark (128 stems), Doubravčice underbark (128 stems)). The stems were described at using stem diameters and landmarks. The gained data were assessed by multidimensional analysis of stem shape diameters and generalized Procrustes analysis. Mean shape averages were calculated for each locality, and the coincidence was tested both for the overbark and underbark sets. For stem shape diameters and Procrustes tangent coordinates, the variability was examined, using the method of principal components analysis. Two most important principal components were diagrammatised and described.

Norway spruce (*Picea abies* [L.] Karst.); stem shape; stem shape diameters; Procrustes coordinates; principal components analysis

INTRODUCTION

Stem shape study is one of the most important subjects of forest mensuration. Traditional forest mensuration describes the stem shape using form quotients, form series, stem profiles, form factors. In recent 25 years, not only "multivariate morphometrics", but also "geometrical methods" have been developed in biology. These methods use a finite number of points, called *landmarks*, for description of an object's shape. A landmark is a point of correspondence on each object that matches between and within populations (Dryden, Mardia, 1998). A shape is intuitively defined as the general geometrical information that remains when the location, scale and rotational effects are filtered out of the object. Two objects have the same shape if they can be translated, rescaled and rotated to each other so that they match exactly, i.e. if the objects are similar. In morphometry, definition of average shape and structure of shape variability is often necessary in a dataset. For that purpose, we mostly use multivariate analysis of stem shape diameters, generalized Procrustes analysis (GPA) (Gower, 1975; Ten Berge 1977, cit. Dryden, Mardia, 1998) and principal components analysis.

MATERIAL AND METHODS

This study compares spruce stem shapes on two localities. The first experimental plot, called Doubravčice, is in the territory of School Training Forest Kostelec nad Černými lesy. The second experimental plot is located within the Protected Landscape Area the Šumava Mts., forest district Lenora, and is marked as Lenora. The investigation material involves 32 spruce stems from each experimental plot that created the base for the reduced analysis.

The sample plot Doubravčice lies on the forest type 2K3 – an acid beech–oak type, elevation 320 m, age 75 years, h_g = 24.4 m, d_g = 21.6 cm, volume of coarse wood was 434.03 m^3/ha. Site index was 28, stand density 0.89.

The sample plot Lenora occupies the forest type 6S1 – fresh spruce-beech stand, elevation 810 m, age 65 years, h_g = 28.9 m, d_g = 25.2 cm, volume of coarse wood was 531.0 m^3/ha. Site index was 34, stand density 0.83.

The selected 32 trees in each set correspond with each other from the viewpoint of relative distance calculated since beginning of both sets according to volumes.

The diameter and height increments were measured in the cut trees after 5, 10, 15 years. Furthermore, a reduced stem analysis was performed. The diameters were examined in a 2m section and at d.b.h. Current diameters were measured with bark and without bark as well as other diameters 5, 10, and 15 years ago. Measurement at experimental plot Lenora was done by Kotek (1973).

Totally 6 sets were formed of these 32 stems: Lenora overbark (32 stems), Lenora underbark (4 x 32 = 128 stems from reduced analysis), Lenora underbark1 (32 stems, the last year before felling), Doubravčice overbark (32 stems), Doubravčice underbark (128 stems) and Doubravčice underbark1 (32 stems, the last year before felling).

The total stem volume in the Lenora overbark set was 24.81 m^3, in the Lenora underbark1 set 22.61 m^3, bark proportion was 8.9%.

The total stem volume in the Doubravčice overbark set was 19.57 m^3, in the Lenora underbark1 set 17.70 m^3, bark proportion was 9.6%.

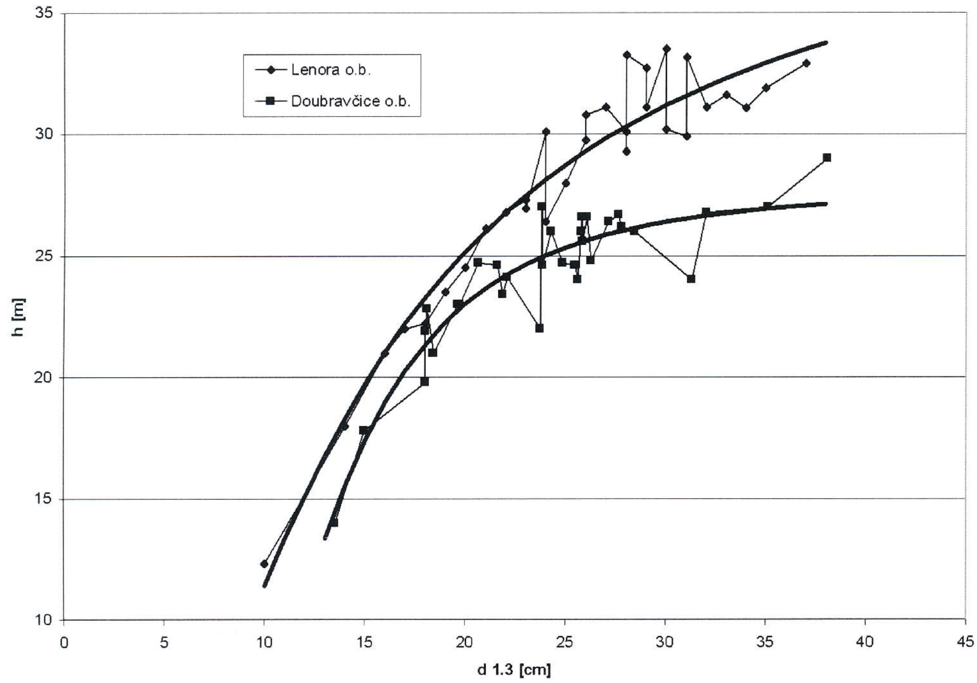

Fig. 1. Height curves for Lenora overbark and Doubravčice overbark sets

If we compare height curves of the Lenora overbark and Doubravčice underbark sets (Fig. 1), we find out that stems in the Lenora overbark set are higher during the whole course of height curve. Height curves are balanced by means of Korf growth function.

The stem can be described as a multidimensional object by means of "stem shape diameters". Thus, the stem shape diameters b_m are the diameters at the relative sections (in this case m = 1/20, 1/10, … 9/10 of the stem height), divided by the stem height (h), therefore $b_m = d_m / h$.

Dividing according to height is in fact size elimination from the object in the sense of intuitive definition of the shape.

Individual stem is therefore taken as a sample from n objects described by m dimensions (stem shape diameters ($b_{i,j}$)). Hence: $\boldsymbol{b}_i = (b_{i,1}, \dots b_{i,m})^{\mathrm{T}}$, $i = 1, \dots, n$.

For this selection, it is possible to set a sample vector for mean values $\hat{\mu}$ given by the following equation:

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{b}_i. \tag{1}$$

Estimation of variance-covariance matrix is ruled by the following equation:

$$\boldsymbol{S} = \frac{1}{n-1}\sum_{i=1}^{n}(\boldsymbol{b}_i - \hat{\mu})(\boldsymbol{b}_i - \hat{\mu})^{\mathrm{T}}. \tag{2}$$

The test of hypothesis that the data are derived from multidimensional normal distribution

In this article, we use a test based on multidimensional skewness ($g_{1,m}$) and kurtosis ($g_{2,m}$), as stated in Meloun, Militký (1998). We test simultaneous validity of hypothesis about symmetry (H_{01} : $g_{1,m} = 0$) and about normality of kurtosis (H_{02} : $g_{2,m} = m\,(m + 2)$) distribution variable of examination. The estimation of sample skewness is given by the following equation:

$$\hat{g}_{1,m} = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}^3, \tag{3}$$

where $d_{ij} = (\boldsymbol{b}_i - \hat{\mu})^T \boldsymbol{S}^{-1}(\boldsymbol{b}_j - \hat{\mu})$ is squared Mahalanobis distance. Considering H_{01} hypothesis valid, then the test statistics

$$U_1 = \frac{n}{6}\hat{g}_{1,m}, \tag{4}$$

has asymptotically chi-square distribution $\chi^2_{m(m+1)(m+2)/6}$.

The estimation of sample kurtosis is given by the following equation:

$$\hat{g}_{2,m} = \frac{1}{n}\sum_{i=1}^{n} d_{ii}^2, \tag{5}$$

Considering H_{02} hypothesis valid, then the test statistics:

$$U_2 = (\hat{g}_{2,m} - g_{2,m}) / (8m((m+2)/n))^{0.5} \tag{6}$$

has asymptotically normal distribution N(0, 1). This approximation can be used providing the following condition is fulfilled:

$$\hat{g}_{2,m} > m(m+2)(n-1)/(n+1). \tag{7}$$

Multivariate test of equality of covariance matrices

For k multivariate populations, the hypothesis of equality of covariance matrices is

$$H_0 : \Sigma_1 = \Sigma_2 = \dots = \Sigma_k.$$

The test $\Sigma_1 = \Sigma_2$ for two groups is treated as a special case by setting $k = 2$. We assume independent samples of size $n_1, n_2, \dots, n_k$ from multivariate normal distribution. To make the test, we calculate

$$M = \frac{|S_1|^{v_1/2}|S_2|^{v_2/2}\ldots|S_k|^{v_k/2}}{|S_p|^{\Sigma_i v_i/2}}, \quad (8)$$

where $v_i = n_i - 1$, S_i is the covariance matrix of the i-th sample, and S_p is the pooled sample covariance matrix. If $S_1 = S_2 = S_p$, then $M = 1$. As the disparity among S_1, S_2 increases, M approaches zero.

Box (1949, 1950, cit. Rencher, 2002) gave χ^2 approximation for the distribution of M. This test is referred as *Box's M-test.* We calculate

$$c_1 = \left[\sum_{i=1}^{k}\frac{1}{v_i} - \frac{1}{\sum_{i=1}^{k} v_i}\right]\left[\frac{2m^3 + 3m - 1}{6(m+1)(k-1)}\right] \quad (9)$$

then

$u = -2(1 - c_1)\ \ln M$ has approximately $\chi^2_{(k-1)m(m+1)/2}$ distribution, where M is defined in (8), and

$$\ln M = \frac{1}{2}\sum_{i=1}^{k} v_i \ln|S_i| - \frac{1}{2}\left(\sum_{i=1}^{k} v_i\right)\ln|S_p|. \quad (10)$$

We reject H_0, if $u > \chi^2_{(k-1)m(m+1)/2}(\alpha)$.

The test of hypothesis that the mean vectors are equal

Consider two independent random samples $\boldsymbol{b}_{x,1}, \ldots, \boldsymbol{b}_{x,n}$ (from sample plot Doubravčice with bark) and $\boldsymbol{b}_{y,1}, \ldots, \boldsymbol{b}_{y,n}$ (from sample plot Lenora with bark). The vectors $\boldsymbol{b}_i = (b_1, \ldots, b_m)^T$, $i = 1, \ldots, n$ are stem shape diameters. In this case $m = 10$ and $n = 32$. We expect that stems from these populations have mean shapes μ_x and μ_y.

The test of hypothesis on mean vectors equality (H_0 : $\mu_x = \mu_y$ versus H_1 : $\mu_x \neq \mu_y$) can be carried out using Hotelling's T^2 two-sample test. Let us use the following test statistics:

$$F_{stat} = \frac{n_1 n_2(n_1 + n_2 - m - 1)}{(n_1 + n_2)(n_1 + n_2 - 2)m}(\mu_x - \mu_y)^T S_p^{-1}(\mu_x - \mu_y), \quad (11)$$

where

$$S_p = \frac{(n_1 - 1)S_1 + (n_2 - 1)S_2}{n_1 + n_2 - 2} \quad (12)$$

is the pooled variance-covariance matrix and S_1 and S_2 are variance-covariance matrices for individual samples.

We can express the squared Mahalanobis distance of equation (11) as

$$d_{xy} = (\mu_x - \mu_y)^T S_p^{-1}(\mu_x - \mu_y) = \sum_{j=1}^{m} s_j^2 / \lambda_j \quad (13)$$

where $s_j = (\mu_x - \mu_y)^T \gamma_j$ are the scores in the direction of the observed group difference, γ_j are eigenvecors of matrix S_p and λ_j are corresponding eigenvalues.

High values of s_j^2/λ_j indicate which directions of shape variability are associated with the difference between the groups.

Provided the null hypothesis is valid, the test statistics F_{stat} has Fisher's distribution with m and $n_1 + n_2 - m - 1$ degrees of freedom. However, this test can be used only in the case of normality of both sets and homogeneity of variance-covariance matrices.

The assumption of normality and equal covariances turned out to be questionable. Therefore, a Monte Carlo test was carried out with the null hypothesis that the groups had equal mean shapes. The data were randomly split into two groups of the same size as the groups in the data, and the test statistic F_{stat} was evaluated for B random permutations $T_1, \ldots, T_B$. The ranking r of the observed test statistic F_{obs} was then used to give the p-value of the test:

$$p - \text{value} = 1 - \frac{r-1}{B+1}.$$

Variability

The *principal component analysis* (PCA) was used to analyse the shape variability. In principal component analysis, we seek to maximize the variance of a linear combination of the variables. The first principal component is the linear combination with maximal variance; we are essentially searching for such a dimension that the observation maximally separates or around which the observation data are spread out. The second principal component is the linear combination with maximal variance in a direction orthogonal to the first principal component, and so on.

The orthogonal eigenvectors of variance-covariance matrix, denoted by γ_i, $i = 1, 2, \ldots, j$, are the *principal components* of variance-covariance matrix with corresponding eigenvalues

$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_j \geq 0,$$

where $j = \min(n - 1, m)$. The principal components are in fact transformed variables and the principal component (PC) score represents transformed objects. PC score for the i-th individual on the j-th principal component is given by

$$s_{ij} = \gamma_j^T(\boldsymbol{b}_i - \hat{\mu}). \quad (14)$$

The standardized PC scores are

$$c_{ij} = \frac{s_{ij}}{\sqrt{\lambda_j}}. \quad (15)$$

RESULTS AND DISCUSSION

All the following tests are calculated for stem shape diameters. In the case of Doubravčice overbark plot, the sample skewness is $\hat{g}_{1,10} = 36.93$. The test statistics U_1 thus equals 197, which is under the critical value of χ^2_{220} (0.05) = 255.60. Sample kurtosis is $\hat{g}_{2,10} = 107.74$. Test statistics $U_2 = -2.24$ and the critical value of standardised normal distribution on the significance level of 0.05 is 1.64. The criterion (7) is not fulfilled, because $\hat{g}_{2,m} <$ 112.73. For calculation of U_2 we use the relation:

$$U_2 = \frac{\hat{g}_{2,m} - \frac{m(m+2)(n+m+1)}{n}}{\sqrt{8m(m+2)/(n-1)}}$$

test statistics U_2 is equal -9.61.

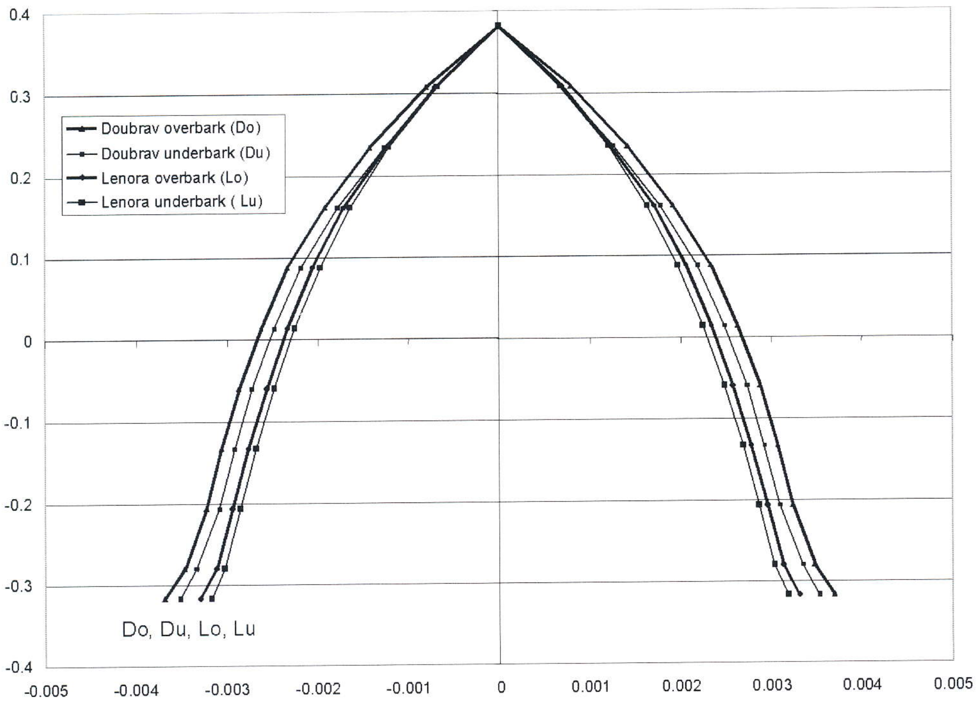

Fig. 2. Stem shapes of the four samples expressed by full Procrustes mean shapes

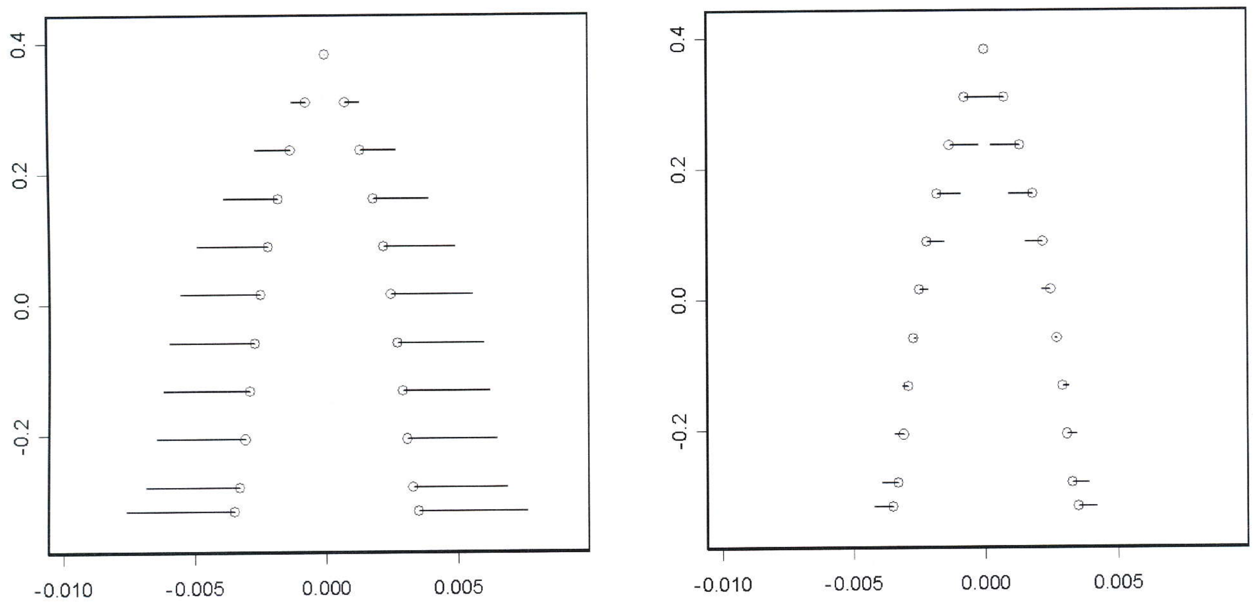

Fig. 3. First two PC with configurations evaluated for 3 standard deviations along each PC from the full Procrustes mean shape in Lenora and Doubravčice overbark plots

In both quantities, skewness and kurtosis, we therefore accept the coincidence with normal distribution.

It also answers to the case of Lenora overbark. Sample skewness is $\hat{g}_{1,10}$ = 48.25 and test statistics U_1 = 257.35. Sample kurtosis is $\hat{g}_{2,10}$ = 115.17 and test statistics $U_2 = -0.881$ The criterion (7) is again fulfilled ($\hat{g}_{2,m}$ > 112.73). In this case, we reject coincidence with the normal distribution ($U_1 > \chi^2_{220}(0.05)$).

The tests carried out indicate a low divergence from multidimensional normal distribution in Lenora overbark set.

We also did the *Box's M-test* for Doubravčice overbark and Lenora overbark sets.

Test statistics

$u = -2(1 - c_1) \ln M = 88.34 > \chi^5_{55}(0.05) = 73.31$, p-value = 0.003. We reject H_0.

The Monte Carlo test was used for assessing the coincidence of mean vectors because of problems with normality in Lenora overbark set and of dependent measurements in Doubravčice underbark and Lenora underbark sets. The coincidence of mean shapes was tested for Doubravčice overbark and Lenora overbark sets and then for Doubravčice underbark and Lenora underbark sets. For each pair of samples, 2,000 random permutations were performed. In both cases p-value = 0.001 and there-

Table 1. F_{stat} partition of Equation (11) for the 10 principal components, Lenora overbark and Doubravčice overbark sets, d_{ij} is the squared Mahalanobis distance

No. of principal component	F_{stat} partition					F_{stat}	d_{ij}
1–5	1.19	0.13	0.30	0.05	0.03		
6–10	0.29	0.31	0.13	1.47	0.69	4.30	3.14

Table 2. F_{stat} partition of Equation (11) for the 10 principal components, Lenora underbark and Doubravčice underbark sets, d_{ij} is the squared Mahalanobis distance

No. of principal component	F_{stat} partition					F_{stat}	d_{ij}
1–5	2.69	0.70	0.00	3.28	0.01		
6–10	0.23	0.52	0.09	4.79	1.13	13.45	2.18

Table 3. Eigenvalues of the variance-covariance matrices of stem shape diameters and Procrustes tangent coordinates from the Lenora and Doubravčice plots, and proportional expression of variability explained by them

<table>
<tr><th rowspan="2">Eigenvalue</th><th colspan="4">Lenora and Doubravčice underbark sets</th><th colspan="4">Lenora and Doubravčice overbark sets</th></tr>
<tr><th colspan="2">Stem shape diameters</th><th colspan="2">Procrustes tangent coordinates</th><th colspan="2">Stem shape diameters</th><th colspan="2">Procrustes tangent coordinates</th></tr>
<tr><td></td><td>$\lambda_j \,.\, 10^{-6}$</td><td>$\lambda_j / \sum_{j=1}^{p} \lambda_j \cdot 100$ (%)</td><td>$\lambda_j \,.\, 10^{-3}$</td><td>$\lambda_j / \sum_{j=1}^{p} \lambda_j \cdot 100$ (%)</td><td>$\lambda_j \,.\, 10^{-6}$</td><td>$\lambda_j / \sum_{j=1}^{p} \lambda_j \cdot 100$ (%)</td><td>$\lambda_j \,.\, 10^{-3}$</td><td>$\lambda_j / \sum_{j=1}^{p} \lambda_j \cdot 100$ (%)</td></tr>
<tr><td>λ_1</td><td>8.04</td><td>91.6</td><td>1.62</td><td>92.0</td><td>8.00</td><td>92.1</td><td>1.46</td><td>92.7</td></tr>
<tr><td>λ_2</td><td>0.431</td><td>4.91</td><td>0.34</td><td>4.04</td><td>0.462</td><td>5.32</td><td>0.34</td><td>5.05</td></tr>
</table>

fore we reject the null hypothesis about coincidence of mean vectors and accept the hypothesis about difference between mean shapes vectors.

Graphically full Procrustes mean shapes for all four sets are illustrated in Fig. 2. Mean shapes of Doubravčice sets are "wider" than those of Lenora sets.

Further question, which principal components differ from each other the most, was solved. Tables 1 and 2 contain the components of test statistics F_{stat}, calculated for individual principal components. Components of Mahalanobis distance s_j^2 / λ_j indicate which directions of shape variability are associated with the difference between the groups. Sets with bark_differ from each other above all in the 9th, 1st and 10th principal component, and sets underbark in the 9th, 4th and 1st principal component.

Graphic effect of the first two principal components is the same in stem shape diameters as well as in Procrustes tangent coordinates. For better illustration and with regard to finished programmes prepared by Dryden (2000), we carried out an analysis of the first two principal components in Procrustes tangent coordinates as shown in Dryden, Mardia (1998); the definitions have been introduced in the same way as in Křepela et al. (2004). Values of the first two eigenvalues for stem shapes diameters and Procrustes tangent coordinates are given in Table 3.

Fig. 3 illustrates graphic effect of the first two principal components in the Lenora and Doubravčice overbark sets. The first two principal components in the Lenora and Doubravčice underbark sets show the same graphic effect. The first principal component explains approximately 92% variability in both sets. It is symmetric pointing across the vertical stem axis. Graphic effect is the same and coincides with the first principal components presented in the previous works of Křepela et al. (2001, 2004, 2005), Křepela (2002) for the Norway spruce and Scotch pine. We explain this component by tree competition.

The second principal component in both sets explains approximately 5% variability. The second PC is asymmetric and has an opposite direction in the bottom section (up to 4/10 of the stem height) and in top section of the stem. Also this effect appeared for spruce and pine during our preceding observations, and we ascribe it to damaged top or wrongly measured stem length.

Fig. 4 presents the Biplot. The angle among position vectors (A…J) of two original variables – stem shape diameters ($\boldsymbol{b}_j$) is inversely proportional to correlation dimension between these two variables. Thus, the lowest dependence is among stem shape diameters in 1/20 of height (A) and 9/10 of height (J). The graph also shows that the original variable A contributes highest to the first component and the original variable marked I to the second principal component. They are stem shape diameters in 8/10 of height.

CONCLUSIONS

In this article we prove that the shape of the spruce stems differs on locality School Training Forest Kostelec

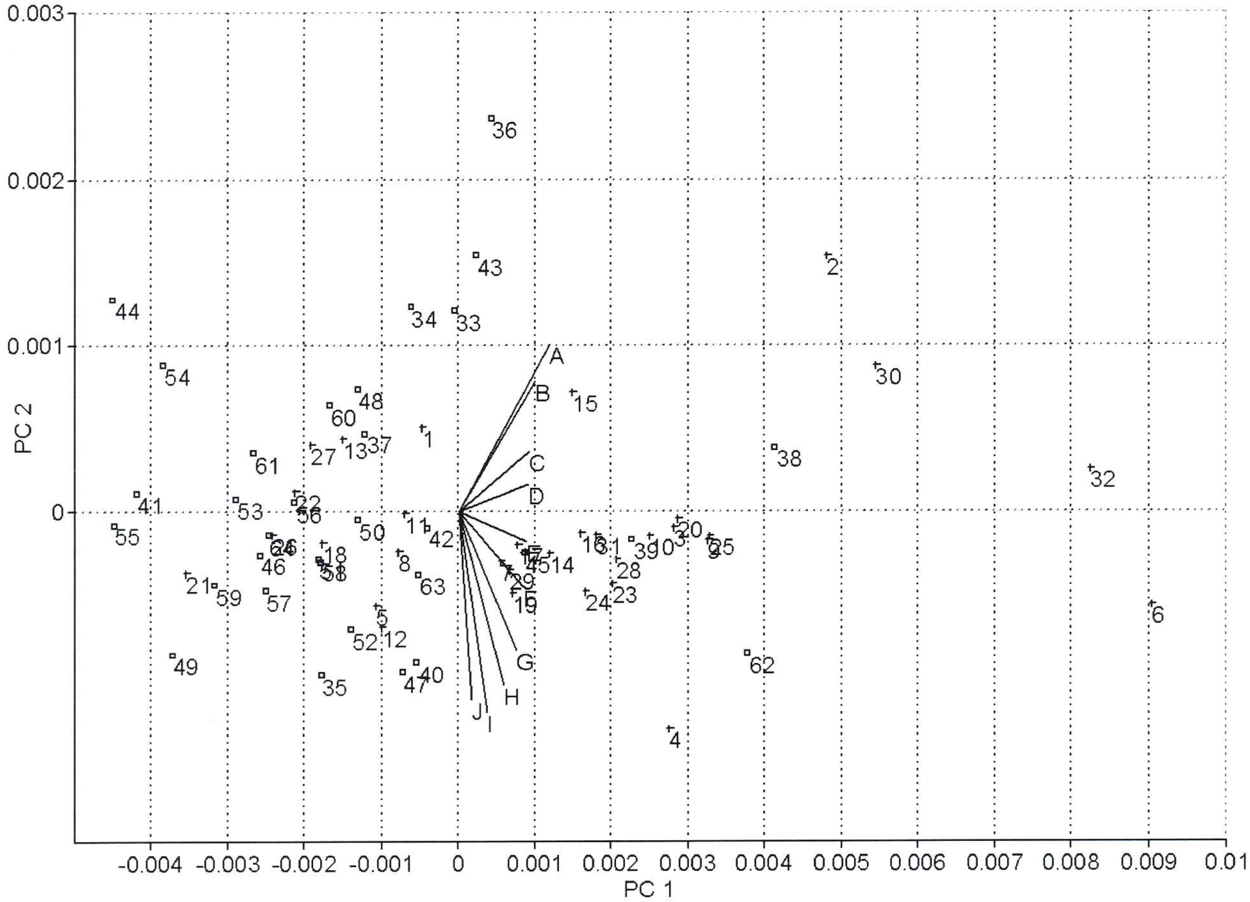

Fig. 4. The Biplot for 64 stems of Lenora overbark sets numbers (1–32) and Doubravčice overbark sets numbers (33–64). Position vectors A … J illustrate stem shape diameters (b_j)

nad Černými lesy from the shape on locality Lenora in the Protected Landscape Area the Šumava Mts. This conclusion applies to the shape of the stems with bark as well as to the shape of the underbark ones. The shape variability was examined by using both Procrustes tangent coordinates and stem shape diameters. First two principal components explain about 97% of variability. Their shape effect was described in the previous paragraph. Sets with bark_differ from each other above all in the 9th, 1st and 10th principal component, and sets underbark in the 9th, 4th and 1st principal component. Dissimilarity of the stem shapes should be considered, for example, at volume and growth models construction.

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Srovnání tvaru kmene smrku ztepilého Norway spruce (*Picea abies* [L.] Karst.) na dvou odlišných lokalitách.

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Práce se zabývá porovnáním tvaru kmene smrku ztepilého (*Picea abies* [L.] Karst.) na dvou lokalitách. První pokusná plocha se nacházela na území ŠLP Kostelec nad Černými lesy, druhá pak na území CHKO Šumava, polesí Lenora. Z obou ploch bylo vybráno celkem 32 kmenů a byla provedena jejich zkrácená analýza, a to 15 let zpětně po pětiletém intervalu. Kmeny byly vybrány jednotně podle své objemové relativní vzdálenosti od počátku v obou souborech. Stromy byly pokáceny, rozřezány po dvoumetrových sekcích a byly proměřeny jejich průměry a podle přeslenů určeny délky.

Po provedení zkrácené analýzy byly kmeny roztříděny do šesti souborů. První dva soubory – Doubravčice s kůrou a Lenora s kůrou – jsou tvořeny 64 kmeny. Druhá dvojice souborů byla pojmenována Doubravčice bez kůry a Lenora bez kůry. Tyto dva soubory tvořilo celkem 256 kmenů. Třetí dvojici – Doubravčice bez kůry 1 a Lenora bez kůry 1 – tvořilo 64 kmenů bez kůry. Tato poslední dvojice sloužila pouze ke stanovení objemu kůry, ne ke zkoumání tvaru kmene.

Tvar chápeme z hlediska „geometrických metod" jako geometrickou informaci o konfiguračních maticích po provedení posunutí, rotaci a přeškálování. Zajištění této procedury umožňují tvarové kmenové průměry nebo Procrustovy souřadnice. V případě tvarových kmenových průměrů nepracujeme s konfiguračními maticemi, ale s průměry v 1/20, 1/10, ..., 9/10 výšky kmene. Tyto průměry jsme vydělili výškou kmene a tím odstranili velikost. Provedení rotace či posunutí nebylo nutné, protože počátek souřadnicové soustavy byl umístěn do středu báze kmene.

Než jsme přistoupili k porovnání středních tvarových vektorů, provedli jsme v souborech s kůrou testy vícerozměrné normality a testy shody variančně-kovariančních matic. Zejména u variančně-kovariančních matic jsme neprokázali jejich shodu, proto jsme museli provést neparametrický Monte Carlo test shody středních tvarových vektorů. Tento test jsme museli provést také v souborech bez kůry, neboť měření v rámci jednoho kmene byla na sobě závislá. Monte Carlo test prokázal, že soubory s kůrou i bez kůry se liší ve svých středních tvarových vektorech získaných z tvarových kmenových průměrů. Dále jsme metodou hlavních komponent provedli rozbor variability. První hlavní komponenta, která vysvětluje přibližně 92 % variability v obou dvojicích souborů, a to pro kmenové tvarové průměry i Procrustovy tangentové souřadnice, má symetrický grafický efekt a spojujeme ji na základě našich předchozích prací s konkurenčním tlakem na jednotlivé stromy. Druhá hlavní komponenta má ve spodní části kmene (do 4/10 výšky) opačný geometrický efekt než ve zbývající části kmene. Vysvětluje přibližně 5 % variability. Druhou hlavní komponentu spojujeme s poškozenými vrcholy nebo špatně změřenými nebo v rámci zkrácené analýzy nesprávně určenými délkami kmene.

Naše studie prokázala, že střední tvary kmene s kůrou i bez kůry jsou na lokalitách Školní lesní podnik Kostelec nad Černými lesy a Lenora odlišné. Tato skutečnost by měla být zohledněna např. při konstrukci objemových a růstových modelů.

smrk ztepilý (*Picea abies* [L.] Karst.); tvar kmene; tvarové kmenové průměry; Procrustovy souřadnice; analýza hlavních komponent

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