DESCRIPTIVE AND NORMATIVE MAPS FROM THE MATHEMATICAL MODELLING POINT OF VIEW*

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Neither definition nor categorization and taxonomy of knowledge mapping are unified in accessible literature and therefore the authors have chosen to start working in this field. A knowledge map is a visual interception of knowledge with the aim of its storage, sharing and development. Weak descriptive knowledge maps may be used for explaining the ideas and concepts connected with operational research models (OR models), as well as for explaining the new knowledge gained with the help of models presented in a well-structured form. Strong descriptive knowledge maps can be used to describe real relations between the objects of the models or real elements in relation to their positioning. In this case the object placing does not describe only its physical position but also, for instance, its economical indexes. Like the normative OR models, normative knowledge maps show the normative solution, or help to find the best, desirable or advisable solution. Considering the means of categorizing knowledge maps from the mathematical modelling point of view, these models together with their properties are presented as a knowledge map.

knowledge map; knowledge map categorization; mathematical model; model construction; algorithm; model solution

INTRODUCTION

Mapping knowledge in its authentic substance has a template in geographical mapping, particularly in military mapping, the roots of which stretch deep into the antique world. The first cartographers, who were already conscious of their limited knowledge, decorated their maps with various dragons and lions in those places, where exact data were absent. The maps documenting the ratio of knowledge in the face of ignorance arose this way, because knowledge in itself rises only on the basis of a successfully solved problem. Geographical maps were static in the principal points, but military maps included some dynamic features because of drawings or other graphic descriptions of the battle or the progress of its stages (pre-battle tactics, battle strategy, possible post-battle situations – many times in various scenarios). Much sooner, 30,000 years ago, the first cave paintings showing how to hunt a wild beast appeared. An unknown hunter codified his knowledge in the dynamic form for the purpose of sharing it with future generations. Knowledge mapping is the visualization of knowledge using a map, i.e. non textual graphical forms which may include a process of problem solving for the purpose of further reading, using, sharing and evolving.

MATERIAL AND METHODS

As mentioned above, a knowledge map is a visual representation of a successfully solved problem, which usually includes the process of solution finding (S t a n f o r d , 2000). The solving process should contain at least four steps of the Simon's problem decomposition, i.e. intelligence activity, design activity, choice activity, and review activity (S i m o n , 1960). G o r d o n (2002) also shows that knowledge maps may be referred to as maps for acquiring knowledge. Knowledge maps are important as knowledge building as well as thinking tools (R o g e r s , 2000).

There are various definitions of the terms 'knowledge map' and 'knowledge mapping'. S t a n f o r d (2001) defines it as follows: "Knowledge mapping quite simply is any visualization of knowledge beyond textual for the purpose of eliciting, codifying, sharing, using and expanding knowledge." Graphic symbols play a key role in each knowledge map; their positions and spatial relationships are mostly expressed with the use of arcs or edges. The knowledge map must show a progression of ideas with relationships beyond their being just spatial. Knowledge maps include conceptual relationships, such as chronological, hierarchical, associative, causal, logical and evaluative (S t a n f o r d , 2001).

Each knowledge map, as a specific type of reality model, for instance, a reality image, simplifies the visualisation of reality.

Typology of models

An Analogue Model is the representation of entities of a system by analogue entities pertaining to the model, e.g. through diagrams (www.problemistics.org). It could be a method of representing a phenomenon of the world,

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often called the 'target system' by another, more understandable or analysable system (en.wikipedia.org).

A Symbolic Model is the representation of entities of a system through symbols. Symbols can be: mathematical, logical or ad-hoc (www.problemistics.org).

Normative (and normatives in general) is a relatively widely used term, particularly in philosophy. A normative principle expresses how things should be and why they should be exactly like they are. For example: People should be substantively equal is a normative value because it is a statement of how things should be.

A normative model is a model of how things ought to work.

A descriptive principle is a generalization describing how all entities of a certain kind do in fact.

A descriptive model is a physical, conceptual or mathematical model describing situations as they are or as they actually appear. It is a model of how things work, but not how they necessarily ought to work.

A prescriptive model is a model used to suggest the required behaviour, properties or features of a proposed system. A prescriptive model is a model giving directives or rules.

RESULTS

Proposed knowledge maps typology

Just as the typology of models is based on model forms, knowledge maps may be divided into two main groups:

- Analogical maps and iconic maps, where the analogy between real objects and symbols, plus their spatial relationships and behaviour are crucial for the map understanding.
- Symbolic maps emphasizing the meaning of symbols, usually mathematical or verbal. Such maps generally don't insist on the symbol position. Elements of these maps are rather abstract (terms, expressions) and relations between them are expressed using mathematical formulas, verbal sentences or phrases.

Another typology of knowledge maps is based on the character of judgment or solution of the (successfully) solved problem (B a r o n , 2004):

- Descriptive maps (weak and strong), describing and simulating the real situation as precisely as possible,
- Normative maps, relating to a typical standard or norm, to optimal solution, or to the best decision,
- Prescriptive maps are used to convey the progress in the process of knowledge creation, transformation and sharing. Usually prescriptive maps are derived from normative maps.

Weak descriptive knowledge maps

Weak descriptive maps describe a real situation using different kinds of symbols and arcs which connect them together. Graph theory models are typical tools for creating this kind of maps. Going through this map helps the user in understanding the problem, increasing his/her level of knowledge of "how to solve" a problem. The relative positions of objects (elements) are unimportant, only the symbols themselves and the quality of their relationships are relevant for map reading and problem solving.

Conceptual maps (Fig. 1) as typical weak descriptive maps are simple and practical knowledge representation tools that allow one to convey complex conceptual messages in a clear, understandable way. They facilitate the organization of terms, concepts and other items mostly in a hierarchical way, wherein the most general concepts lie at the root of the tree. Moving down the structure we may replace them with the more specific ones. A conceptual map is a diagram showing the relationships between concepts. Concepts are connected with labelled arrows, in a downward-branching hierarchical structure. The relationship between concepts is articulated in linking phrases, e.g., "gives rise to", "results in", "is required by," or "contributes to" (http://en.wikipedia.org). Passing through this map means following the flows not only from general to specific but also from abstract to concrete.

Decision trees, flow charts representing algorithm progresses and network diagrams for strategy implementation are only just a few representatives of this kind of maps in the field of Operations Research and Management Science.

Strong descriptive knowledge maps

Not only objects, symbols or texts are important for this kind of knowledge map. To be a knowledge map of this type the item must use spatial relationships to elicit, share and codify knowledge (S t a n f o r d , 2001). Such knowledge map must show a progression of ideas with relationships beyond their being just spatial. Knowledge maps include conceptual relationships, such as chronological, hierarchical, associative, causal, logical and evaluative. Geographical maps are typical platforms for strong descriptive maps drawing. Objects with properties and their spatial relations are mapped using homomorphic projection and a good quantitative (or sometimes qualitative) metric is needed for object distances measurement. As a quantitative expression of distance the closest distance between two nearest points of objects is considered. Not only the distance units can be used for such measurements - in special types of strong descriptive maps also costs, weights or points could be used.

Considering the distance ad its measurement, three types of spatial relationships are defined: proximity, adjacency, and containment (Fig. 2).

The rules for relationship types establishing:

- **Proximity**: Distance between objects *Ui* and *Uj* is non-zero, positive but small, i.e., where $V(Ui, Uj) \in (0; M)$ is the upper limit of distance, where the objects still interact.
- **Adjacency**: Distance between objects *Ui* and *Uj* is equal to zero, both objects have a common interface, i.e. $V(Ui, Uj) = 0$

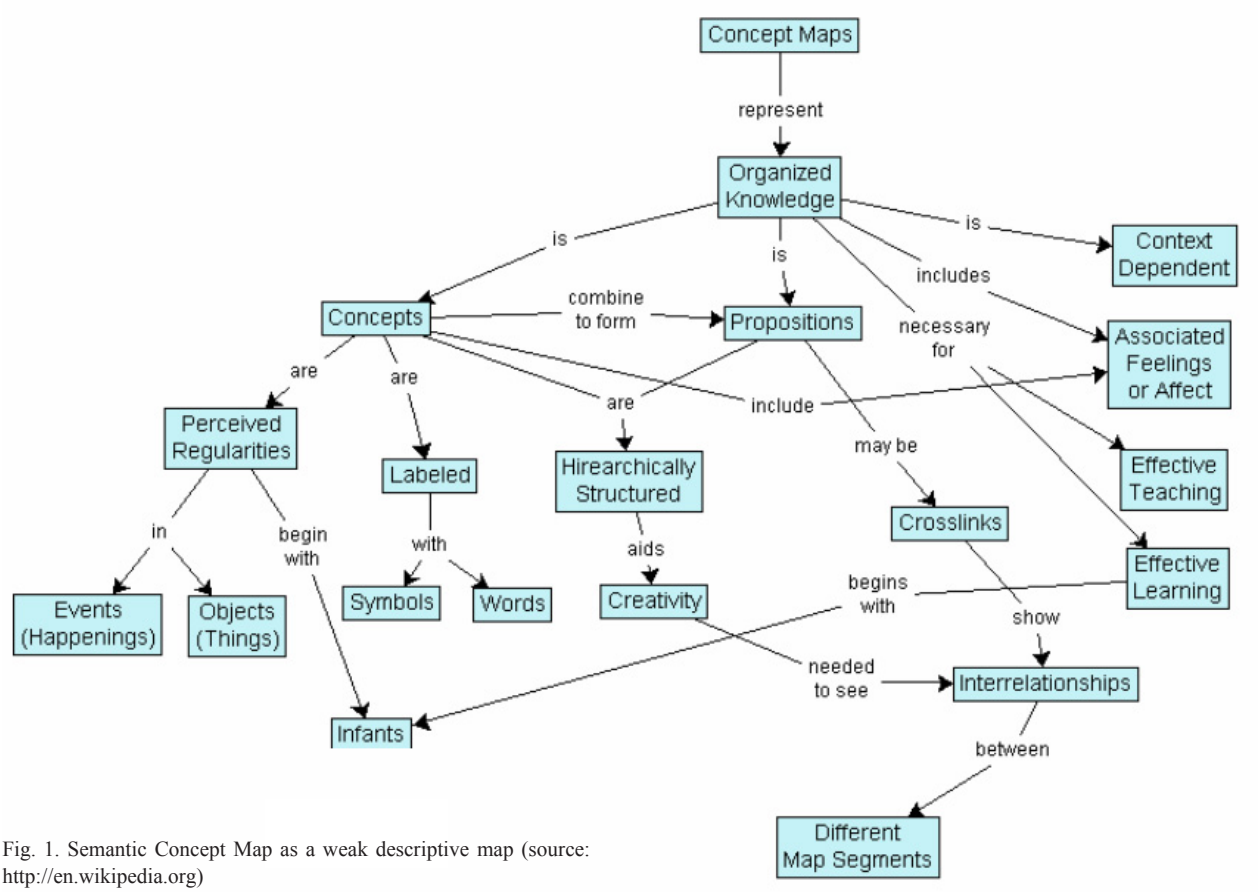


Fig. 1. Semantic Concept Map as a weak descriptive map (source: http://en.wikipedia.org)

Fig. 2. Spatial relationships types in strong descriptive maps

- **Containment**: Distance between objects *Ui* and *Uj* is negative. i.e. (V (*Ui, Uj*) < 0). Objects can but need not have a common interface. According to the existence or non-existence of a common interface we define either partial or full containment.

Strong descriptive maps should be usually expressed using directed graphs or the direction in another type of map must be apparent (for instance Gantt Charts – Fig. 10).

Normative knowledge maps

In this case, the aim of the knowledge map is to introduce the approach of how to reach the target (solution), or of how to reach the comparative norm.

Strategy maps cover the major part of this knowledge map type. Strategy maps are a way of providing a macro view of an organization's strategy, and provide it with a language in which they can describe their strategy, prior to constructing metrics to evaluate the performance against their strategy (en.wikpedia.org).

Strategy knowledge mapping is a technique of creation and use of graphical interpretation of situation in an organization or any other system. A strategy map is a diagram that describes how an organization creates a value by connecting strategic objectives. It describes organizations trends, main streams of effort (mission and vision statements) and targeting (ways for reaching the norm). A good example of normativity of strategic maps can be shown by the example of the so-called Vee map (Fig. 4). As its name suggests, the Vee map is of the letter "V" shape. It is a way of exploring the tension between the theory and the method and using this to gain and retain knowledge. The Vee map follows two axes extending down from the top to form a point at which they join at the bottom. The theory or concepts follow one axis and the methods or how-to follow the other. The problem statements or questions for examination are fed or funnelled down the centre between the two axes and eventually the assessment against each will bring the examiners to their conclusions or solutions (S t a n f o r d , 2001).

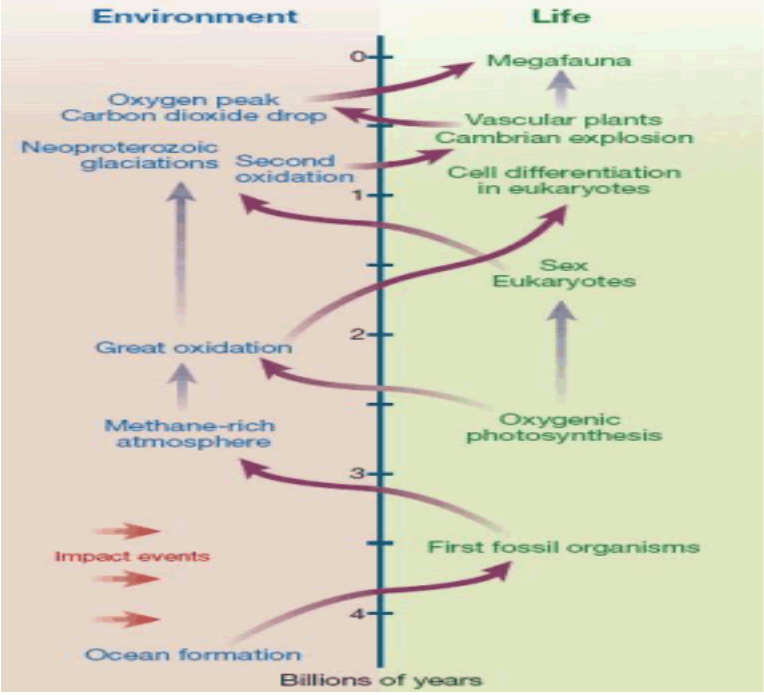


Fig. 3. Earth Evolution Chart as a strong descriptive knowledge map (source: Earth System Modelling Group, http://tracer.env.uea.ac.uk/esmg/)

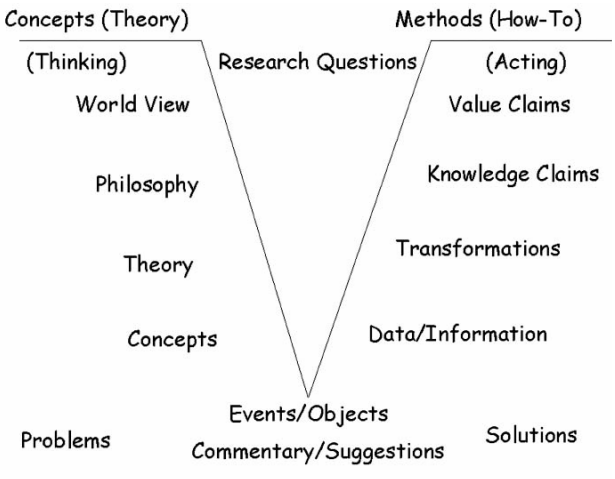


Fig. 4. Basic Vee Map as a representative of normative knowledge maps (source: S t a n f o r d , 2001)

Prescriptive knowledge maps

General prescriptive map is a term widely used in the field of geographical information systems (GIS). This type of map is also useful as a basis for examining performance of the supply chain. Visualization of mathematical algorithms or simulation models using flow charts should be a good case of a prescriptive knowledge map.

Complexity of knowledge maps

The term complexity of knowledge maps is also derived from the term system complexity. It closely depends on the number of connected (joint) elements in the map. The complexity of a map may be expressed using the formula:

$$C = \frac{N}{\binom{n}{2}}$$

where: N – number of existing connections between objects
n – number of objects (elements) in the map

In contrast to the complex system, more complex maps do not necessarily describe solutions of a more complicated problem. Generally there exists a threshold, depending on the type of the visualised problem. Beyond this point the map will no longer be transparent. Maps with complexity equal to one (C = 1) usually provide neither knowledge nor information.

DISCUSSION

Knowledge maps and mathematical models - when and why use them?

The answer to this question can be found using a conceptual map describing the relation between solved problems, existing models and the process of application. This map can be characterized as a weak descriptive map showing the main steps of process solving. B r i n k m a n n (2005) shows this type of knowledge maps as a tool to build structures in mathematics.

Let us consider the Leontief Input – Output Model as a typical example of knowledge maps. The first map (Fig. 5) represents the selection of the proper model type. This map is a normative map, because it shows which model must be used for solving different problems. The

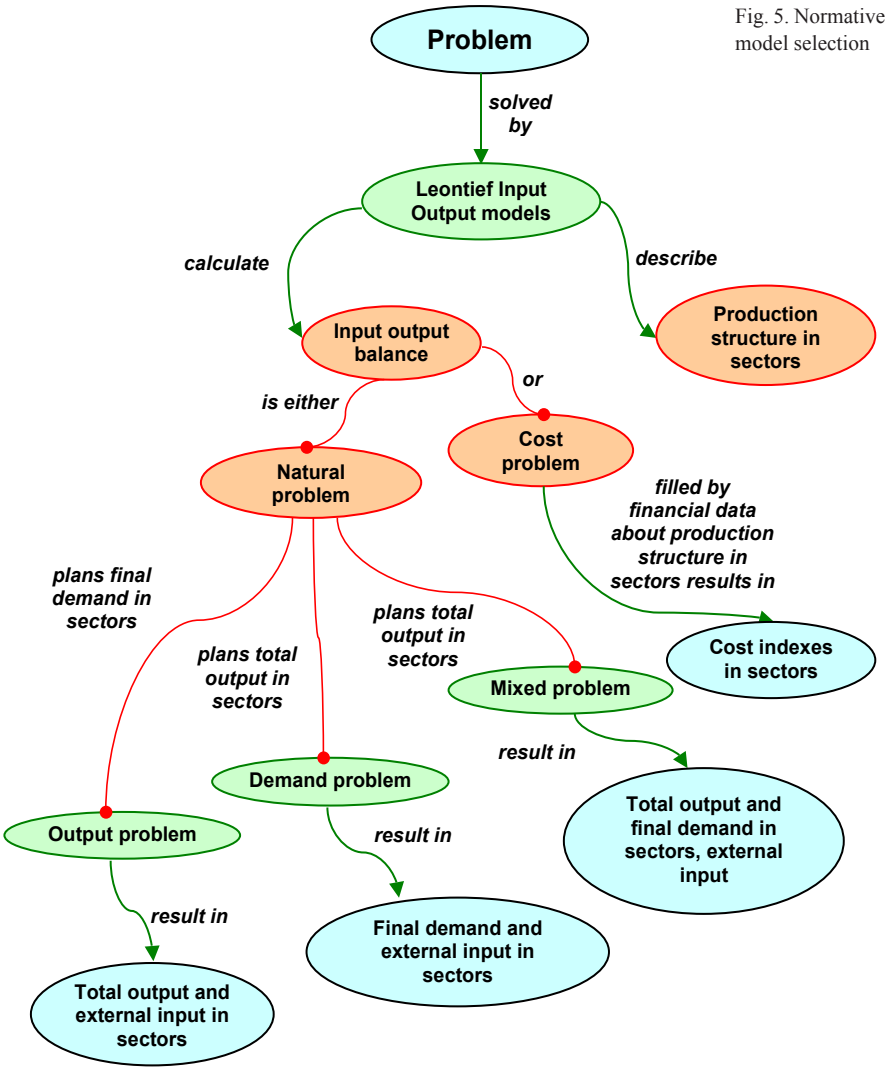


Fig. 5. Normative knowledge map for the best model selection

second map (Fig. 6) shows the data necessary for the application of this type of model and relations between the model features; it is a weak descriptive knowledge map.

Simulation models

The question may be asked: which knowledge map may be considered as a standard simulation model? Usually a standard simulation model is drawn (designed) as a flow chart (Fig. 7) leading to a weak descriptive map. Only object symbols and the quality of relations between them are important for the model simulation. The object placement is important only for objects consequence description and it is impossible to set some metric for this placement. But this kind of map describes the procedure, how the model should work. The flow chart in this case is both a descriptive and a prescriptive map

Decision tree

A decision tree is a graphical form of a decision model and it may be regarded as a weak descriptive knowledge map describing decision situations, possible decision alternatives, and states of nature and sequence of these elements. Adding rules for the best alternative selection leads to a normative knowledge map visualizing the normative decision (Fig. 8, where c_i – criteria, v_i – possible alternatives, Ai – best alternative selected, F, Φ – criteria functions).

Knowledge maps in project management

A conventional project management model is, from the mathematical point of view, a graph theory model or network model. Usually, the project tasks are drawn with the use of the nodes in this chart and the relationships between the nodes are expressed using arcs (Activity on Node graphs). As mentioned before, the network model (sometimes called a PERT Chart) is a weak descriptive knowledge map. More sophisticated methods can be used for expressing the knowledge flow and transformation during the project tasks progress. One of them is a WBS (Work Breakdown Structure) chart (Fig. 9), which describes the project tasks hierarchy without time information. This chart is also a weak descriptive map.

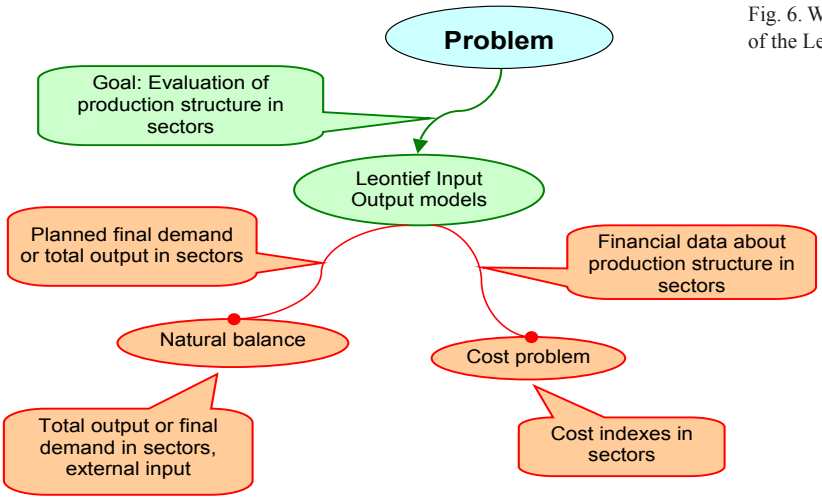


Fig. 6. Weak descriptive knowledge map of the Leontief models

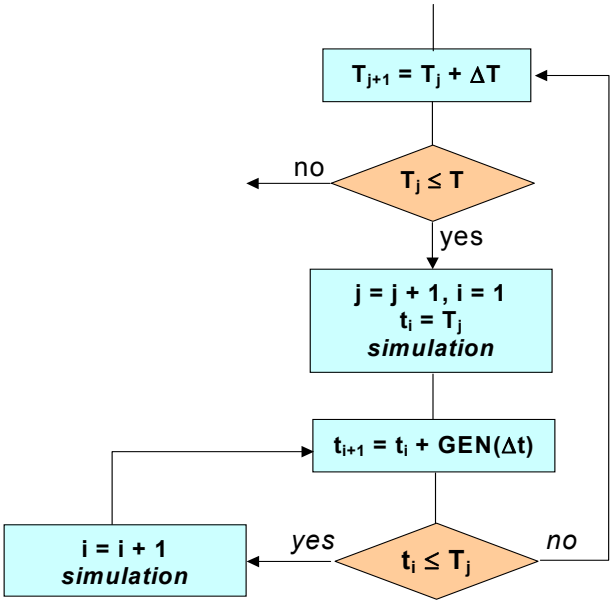


Fig. 7. Example of a simulation model flow chart (weak descriptive or prescriptive map)

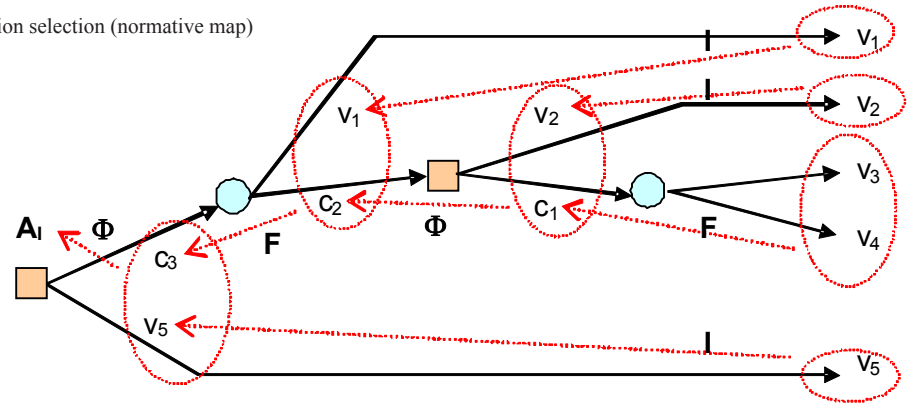
The Gantt Chart (Fig. 10) places every task on a specific date and displays the tasks dependencies exactly on the time scale, and is therefore a strong descriptive map. Generally, usage of timescales moves this kind of knowledge map one level higher – into the group of strong descriptive maps. Tools for the tracking project progress and comparison of the task finish dates transform this mapping into a normative knowledge map.

CONCLUSION

This paper suggests a new type of knowledge map classification based on OR models features. This approach arises from the idea that knowledge and application of the Operational Research models can be read as a graphical representation using different types of knowledge maps.

Weak descriptive knowledge maps may be used for explaining the ideas and concepts connected with OR models, as well as for explaining the new knowledge gained with the models, in a well-structured form.

Fig. 8. Decision tree with the best decision selection (normative map)



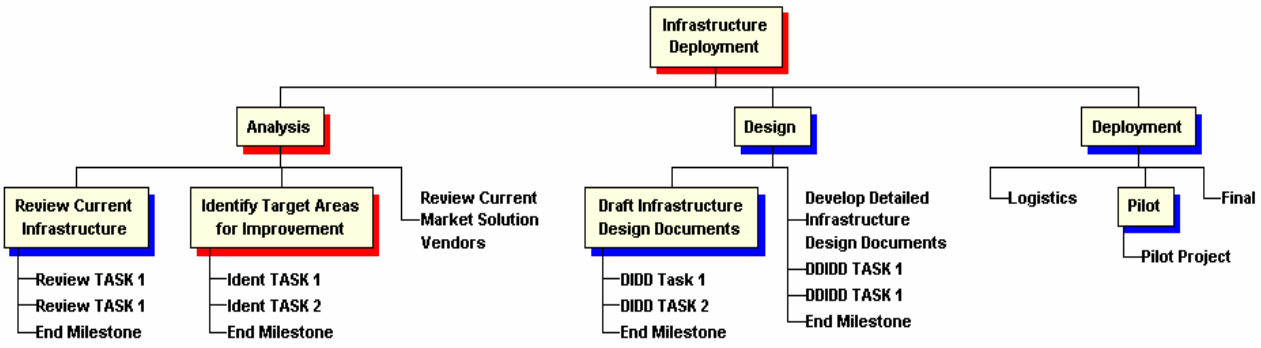


Fig. 9. WBS Chart (weak descriptive map)

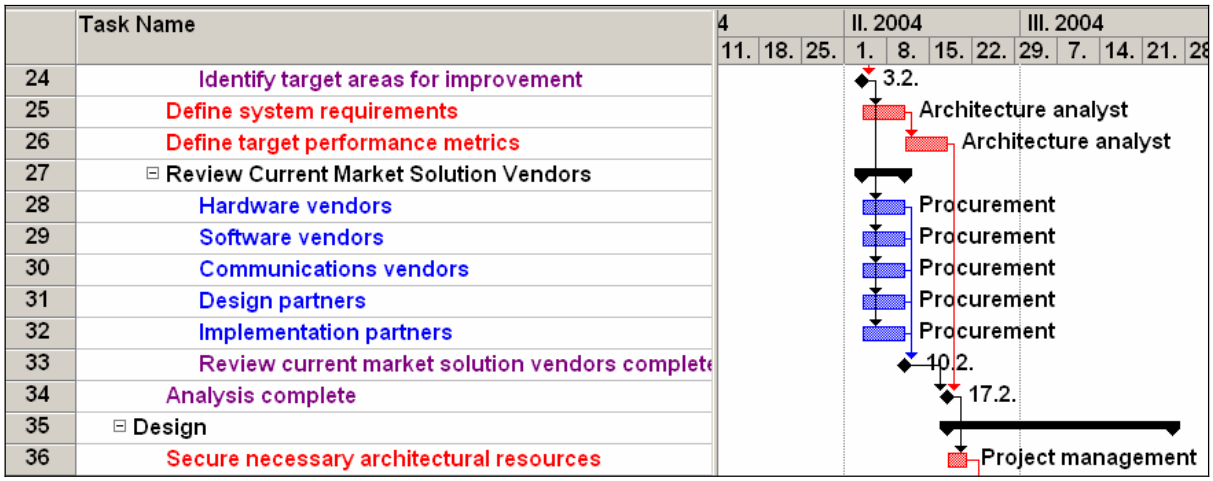


Fig. 10. Gantt Chart (strong descriptive map)

Strong descriptive knowledge maps can serve to describe real relations between the objects of the models or real elements in relation to their positioning. In this case the object placing does not describe only its physical position but also, for instance, its economical indexes.

Like the normative OR models, the normative knowledge maps show the normative solution, or help to find the best, desirable or advisable solution.

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Deskriptivní a normativní znalostní mapy z pohledu matematického modelování.

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Znalostní mapa jako jeden z možných prostředků vizualizace znalostí, resp. vizualizace procesu tvorby, sdílení a transformace znalostí, je v současné době nástrojem velmi frekventovaným. Ne vždy je však užívána vhodně a jen málokdy je užívána systémově. Definice v literatuře se obvykle omezují na vymezení znalostní mapy jako vizualizací znalostí vyřešeného problému zahrnující i proces řešení. Podrobný rozbor, jednotná typologie či kritický pohled zatím nebyl publikován.

Cílem první části příspěvku je definovat, která z grafických reprezentací je opravdu znalostní mapou a jak je možno znalostní mapy kategorizovat, resp. třídit. Znalost vzniká pouze v konfrontaci s problémem, resp. s jeho úspěšným vyřešením – proto také čtení znalostní mapy musí vyústit v úspěšné vyřešení problému. Autoři se ve své práci věnují systémové analýze a matematickému modelování, proto se snaží do znalostního mapování vnést řád především z pohledu modelování, resp. se pokoušejí aplikovat některé všeobecně známé pojmy z oblasti modelování také do znalostního mapování. Jedná se především o pojmy deskriptivní, normativní a preskriptivní. Tyto termíny charakterizující modely z hlediska jejich vztahu k metodě (deskriptivní), cíli (normativní) a postupu (preskriptivní) mají své opodstatnění i ve znalostním mapování. Deskriptivní mapy popisují cestu (resp. cesty) získávání či rozvoje znalostí. Je možné je dále rozdělit na slabě a silně deskriptivní, a to podle možnosti či nemožnosti (účelnosti či neúčelnosti) zavedení metriky mezi objekty na daném nosiči znalostní mapy. Normativní mapy znázorňují, jak postupovat pro dosažení komparativní normy, vzoru, známého optimálního stavu. Preskriptivní mapy se věnují výhradně postupu „step by step“ a jsou vhodné pro vizualizaci znalosti coby schopnosti aplikovat matematický algoritmus.

Druhá část příspěvku je věnována možnostem, jak nahlížet na matematický model jako na znalostní mapu. Ne každý model je znalostní mapou, je však možné vhodnou vizualizací z většiny z nich (resp. z procesu jejich řešení a analýzy výsledků) tuto mapu udělat. Maticové strukturní modely je možné zobrazit podle účelu jak mapou deskriptivní, tak normativní. Rovněž tak modely vícekriteriální analýzy variant je možno nahlížet oběma těmito způsoby. Simulační modely v reprezentaci vývojovým diagramem chápeme jako mapy preskriptivní. Rozsáhlé pole pro znalostní modelování tvoří mapování rozvoje znalostí na poli řízení projektů, resp. projektového managementu. Různé zobrazení projektu vede na jedné straně k mapám slabě deskriptivním (síťový graf a jeho modifikace, WBS diagram jako diagram hierarchické struktury úkolů v projektu), ve kterých je rozvoj znalosti dán zobrazením souvztažností mezi jednotlivými úkoly projektu, jejich souhrny a subprojekty, ve výsledku vedoucí k pochopení složitosti projektového díla. Zahrnutí faktoru času, milníků a ukotvení úkolů vede k mapám silně deskriptivním (např. Ganttův diagram) umožňujícím navíc posoudit časový rámec celého projektu, jakož i časovou náročnost jeho dílčích etap.

znalostní mapa; kategorizace znalostních map; matematický model; vytvoření modelu; algoritmus; řešení modelu

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