METRICS IN KNOWLEDGE MAPS*

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Relation between solution of a problem and its relevant description by the knowledge map is studied. Knowledge maps are used for visualisation and/or description of procedures and algorithms in education and knowledge management. Definition of knowledge map based on union of three objects Support S, Graphics G and Text T is proposed. Special measures are involved into knowledge map: definitions of partial and full formalisation, mapping among graphical objects in groups of knowledge maps, ordering, good ordering. Definitions of layers and series of knowledge maps are given. Examples of robust and soft Supports and Graphics in knowledge maps.

knowledge map; support of knowledge map; graphics; layout of knowledge maps; series of knowledge maps

INTRODUCTION

Knowledge increasingly becomes the key strategic resource of the future development. Although transfer of knowledge within organizational structures can be realized through many different ways, these all have one common base: education and training. It is impossible to transfer and share knowledge among employees without having well developed and well functioning educational systems in organization.

Following text is devoted to one rather special tool for support of mutual share of knowledge among people: it is *knowledge map*. The term is used in many different ways and directly concerns the process of visualisation within common share of knowledge, see Mentzas et al. (2003), Ricardo et al. (2007). The main role of this support is to facilitate the creation, sharing and application of knowledge in schools, companies and/or daily life. In an effort to provide a framework that is generic enough to support individuals and at the same time to facilitate the transfer of needed knowledge, special drawings and graphical schemas are elaborated. A knowledge map can describe an elementary knowledge transfer as well as the whole knowledge life cycle, from knowledge acquisition to knowledge use. The knowledge map is supplemented by procedures which make possible to the user to enter relevant document bases, topics, sources, narrative summaries, higher level descriptions, search and retrieval services available, access to multiplatform, heterogeneous sources, including internet and intranet sites, file servers, databases, popular proprietary formats and legacy information systems, etc.

MATERIAL AND METHODS

Knowledge maps are used in the education and management of knowledge as a methodical means for the visualisation and description of algorithms, methods and procedures when solving scientific problems, as well as didactic tools in the study literature. The knowledge map therefore acts here as a peculiar graphics formation in many images and shapes. One and the same specialised or pedagogic interpretation can be illustrated by means of this graphics environment by considerably different graphics means which, nevertheless, lead towards the same aim: by means of graphics objects, symbols and textual descriptions to show and make accessible to the reader the contents of specialised information, express its structure, indicate mutual connections between parts of specialised text, to show time, spatial or other links between objects and actions, etc. The considerable variability of these possibilities enables authors to insert into the knowledge maps construction their own specialised creative ability, pedagogic competence, experience and also to adapt this graphics environment for the needs of the target groups, i.e. to modify the knowledge maps according to the needs from the viewpoint of their use in time and space.

Although we occasionally come across the definition of the knowledge map in specialised literature, this concerns an explanation of this term, rather than its definition. The term knowledge maps is usually understood intuitively and this sometime leads to situations in which we cannot make a decision as to whether a specific object is or is not a knowledge map. The aim of this paper is twofold: a) to formulate a definition of the term knowledge map, which comes from the object character of this formation, b) to introduce metrics into this object – this will enable classification and sorting out of knowledge maps.

Methodology of this work is based on the following: a) formulation of the definition of the knowledge map and the corresponding expressions and terms, b) analysis of the common properties shown by different knowledge maps presented in specialised literature, and c) selection of parameters for the metrics that would be suitable for

* The paper is supported by the grant project of the Ministry of Education of the Czech Republic No. MSM6046070904 – "Information and knowledge support of strategic management".

measuring of properties of the newly defined knowledge map.

RESULTS

1. Word "map" in the expression "knowledge map"

What is meant by words "knowledge" and "map" in the term "knowledge map"? This term obviously consists of two words: "knowledge" and "map". The term "*map*": from the name itself it is clear that it concerns a picture (from the Latin word mappa = napkin, we can assume it means a picture which is mapped in the plane). Generally, the term map is understood to be a presentation of terrain formations by means of agreed standard symbols on the plane. However, the term has expanded, so that the geographic formations are now depicted even in spatial, 3-dimensional maps, or on globes. We also depict other spatial formations, such as a starry sky or virtual projects. The basic common elements that can be identified in most maps are as follows:

a) established system of coordinates (e.g. latitudes, meridians),
b) established metrics (the map scale, cardinal points N, S, E, W, Greenwich, equator, divisions into degrees or hours, contour lines, etc),
c) set of agreed standard symbols (signs representing objects such as towns, streams, transport communications, airports, monuments, colours designating plains or mountain regions, etc. – certain special or less known symbols are shown in the attached legend).

2. Word "knowledge" in the expression "knowledge map"

The term "*knowledge*" refers to the fact that the knowledge map concerns knowledge, particularly its transfer. In the definition of the term "knowledge map" it is necessary to precisely qualify the term "knowledge" in order to distinguish it from the terms of "data" and "information"; to this problem see Havlíček (2006), Havlíček et al. (2006a, b, c).

Most definitions of knowledge put direct relation between two categories, namely information and problem. Between the notions of "knowledge" and "problem" there is a clear and causal interdependence. For the explanation of the term "knowledge" used in the phrase "knowledge map" we can define: *"Knowledge is information which is used to solve successfully a problem and can be shared with others to solve or facilitate the solution of similar problems"*.

Knowledge cannot be separated from the problem: if there is no problem, there is no knowledge. Therefore, when working with knowledge and using it in the knowledge map, we will always require to have the following objects at our disposal: a) an identified problem, b) successful solution of this problem described in a suitable form (e.g. text form, algorithms, multimedia, etc.).

3. Definition of knowledge map

3.1. Definition

By the knowledge map K we understand a union of three objects: the Support S, Graphics G and Text T:

$$K = \cup\{S, G, T\},$$

where the Support S is a domain of plane or space, Graphics G is a set of plane or spatial geometric objects and Text T is a set of alphanumerical signs.

3.2. Notes to definition

1. *The Support* S of the knowledge map defined in the Definition 3.1. (hereafter Support) can be a plane formation, such as a rectangle, square, polygon, circle, ellipse, etc. The Support can be even an entire plane. Similarly, in the 3-dimensional space the Support can be the whole space or its part. The boundaries of areas can be marked or unmarked. In knowledge maps the plane Supports occur most frequently.
2. *Graphics G* (hereafter Graphics) is a part of mathematical Theory of Graphs which deals (apart from other things) with the properties of shape and position of geometrical formations. Graphics, as part of the union of objects of the knowledge map in the Definition 3.1., can represent a set of very diverse geometrical formations that can be regular (e.g. shapes from a developmental diagram, lines, broken lines, arrows, arches, etc), irregular (drawings and sketches, clumps, smileys, close-ups of pictures, iconic models of objects, etc.), or combinations of regular and irregular formations.
3. *Text T* (hereafter Text) in a knowledge map – e.g. text chains and/or alphabetic characters – can be attached to a geometrical object of Graphics G. In such cases we say that it is *incidental with Graphics G*. If it is not so, we call it a *free text chain*.
4. Knowledge map within the meaning of the Definition 3.1. is a structured formation and we can subdivide it into three disjunctive sets of these elements: (1) Support, (2) elements forming the Graphics of a knowledge map with incidental text chains, and (3) free text chains.

4. Formalisation in knowledge maps

4.1. Definition

1) We say that the *Support S is formalised* when it has a defined system of coordinates with a unit and orientation. When it is not so, we say that the Support S is empty or that it is not formalised.
2) Similarly, the *Graphics G is formalised* when it consists of the elements of the Theory of graph. In the opposite case we say that the Graphics G is not formalised.
3) The knowledge map is *partially formalised* when it contains formalised Support S or formalised Graphics G. The knowledge map is *fully formalised*, or only formalised when it contains alongside both formalised Support S and formalised Graphics G. When the Sup-

port S and Graphics G of knowledge map are not formalised, we say that the knowledge map is not formalised.

4.2. Notes to definition

1. Formalisation of the Support can be robust or soft. An example of *robust formalisation* is the standard system of Cartesian coordinates on the plane or in the space. This system is marked out on the Support while other objects of the Graphics and Text are positioned within its coordinates. Likewise, the systems of coordinates of standard cartographic maps, cadastral plans and similar documents also belong to the robust formalisation. The Supports, which are formalised by a cartographic map, cadastral plan or by similar documents, are robustly formalised.
2. *Soft formalisation* is based on delimiting the coordinates on the support by other means: the area of the support can be divided/separated into quadrants or octants or other shapes of separate spaces, the diagonal in a rectangle can delimit the positional properties of objects (e.g. to mark the association of objects with two extreme processes or, conversely, mark their indifference to these processes). Other graphic schemata, such are for example "V-maps" are also used. The supports with soft formalisation can also include photographs (in so far that they enable identification of the position, distance and orientation – e.g. by means of aerial surveying/photogrammetry), GPS images, and other similar documents.
3. Formalisation of Graphics makes it possible, during the construction of a knowledge map, to utilize a mathematical Theory of graph, which has been developed for a multitude of applications and comprises a number of standard terms and symbols. The names and properties of different types of graphs (such as a knot, isolated knot, edge, orientation, cycle, tree, net, road, chain, etc.) can be taken in their entirety, including the symbols which are normally used in applications (Hamilton outline, cycle, CPM, application in Petri nets, neuron networks, etc.).

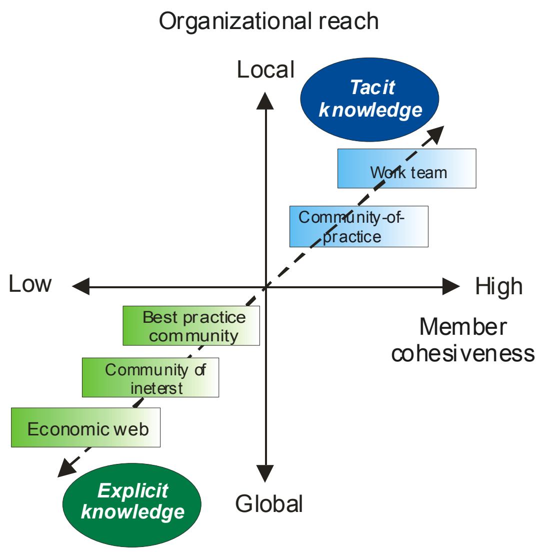

Fig. 1. Example of knowledge map with formalised Support – formalization by Cartesians

4. A formalised knowledge map can utilize standard elements of several specialised disciplines. As an example we can use the solution of a problem described by a knowledge map, whose support is a cartographic map and whose Graphics is formed by objects taken from the theory of graphs. In such a map we can use a number of standard symbols and objects, and thus reduce to a minimum the free text chains, as well as the text chains which are incidental with Graphics.

4.3. Example

Knowledge map on Fig. 1 has formalised Support by Cartesians. Graphics is also formalised. Thus, the map is fully formalised. Text contains both incidental and free chains.

5. Layer and series of knowledge maps

A knowledge map represents a solution of a problem. Similarly to the case in which complex problems are divided into smaller and simpler problems and the solution of the original problem is transformed into a gradual or parallel solution of partial problems, the knowledge map can also be broken up into smaller parts in which the solution of problems is achieved in parts.

Solution of a problem can thus be defined by several knowledge maps. Consistent with the assumption of an unambiguous agreement between the solution of a problem and knowledge (represented here by a knowledge map) we will define two groups of knowledge maps describing and visualising the solution of a problem. For this purpose we will introduce the term of mapping among graphic objects of groups of knowledge maps.

5.1. Definition

Let $\{K_i\} = \{K_1, K_2, \ldots, K_m\}$ be knowledge maps with Graphics $\{G_i\} = \{G_1, G_2, \ldots, G_m\}$ and let each Graphics G_i contains graphical objects g_{ij}, e.g.

$$G_i = \{g_{i1}, g_{i2}, \ldots, g_{ij}\},$$

where $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$.

1) If there exists an unequivocal bijective mapping between the elements of $g_{ij} \in \{G_i\}$, we say that the knowledge maps $K_1, K_2, \ldots, K_m$ form a *layer* $L(K_i)$ of knowledge maps.
2) If there exists a mapping between at least one pair of graphical objects

$$\{g_{i-1,j}, g_{ij}\} \in \{G_{i-1}, G_i\},\ i = 2, 3, \ldots, m;\ j = 1, 2, \ldots, n,$$

we say that the knowledge maps $K_1, K_2, \ldots, K_m$ form a *series* $S(K_i)$ of knowledge maps.

5.2. Notes to definition

1. There is no requirement in the definition of the layer of knowledge maps $L(K_i)$ for the Supports to match each other. Thus, generally, each map in the layer can have a different Support.
2. If we placed all maps of the layer on top of each other, the objects of their Graphics would "cover one other" in a sense that their graphic objects clearly correspond with each other: the number of the graphical objects in each map is identical. If we numbered these objects, we would find that in each map there would be corresponding objects with identical numbers. However, the positions of the objects on the Support do not have to correspond with each other.
3. Bijective mapping occurs in knowledge maps which describe the solution of a problem from the point of view of different aspects: e.g. they initially characterise objects from the viewpoint of their names (the first layer, metrics of the zero order), followed by the characteristics of the objects from the viewpoint of their properties (second layer, metrics of the first order, concerns data), followed by the description of the mutual relationships between the objects (third layer, metrics of the second order, concerns information), followed by the description of an active or passive participation of the elements in the solution of the problem (fourth layer, metrics of the third order, concerns elementary knowledge), etc. (Havlíček et al., 2006a).
4. The layer $L(K_i)$ is represented as a sequence of knowledge maps, in which the order in the sequence is expressed by the order in the layer. Virtual representation in multimedia makes it possible to present the layer of knowledge maps as a real visible layer of sheets.
5. Mapping between pairs of elements in the series $S(K_i)$ is not unequivocal, which means that between the pairs of maps K_i in the series $S(K_i)$ there can be mapping "in" as well as mapping "on" the set of elements {gij}. Therefore, in each pair of maps $(K_{i-1}, K_i) \in \{K_i\}$, there must be a correspondence between at least one element from each map.
6. A typical example of a series of knowledge maps is visualisation of the algorithm of a problem solution, in which each map of a series always contains an object which marks out the start of a solution. This object is marked out in each map of the series. Similarly, in each map of the series there can be repeatedly represented objects serving as key points, which make it easier for the user to find orientation in the progress of the solution.
7. In managerial practice we often solve complicated and complex problems whose solution is characterised by a complicated structured methodology and algorithms. When describing the solution of such complex systems we can combine the layers and series of the knowledge maps.

5.3. Example

Fig. 2 depicts symbolically the layout of three knowledge maps. Each map in the layout has the same number of graphical objects. Among objects there exists bijective mapping. Note that the spatial placement of objects on the Supports of maps is not the same.

Fig. 3 depicts symbolically the series of three knowledge maps. Between each pair of maps in the series there is at least one mapping between graphical objects of their Graphics.

Fig. 4 depicts two maps of the series of knowledge maps. The "blue" is the first map in the sequence. The "orange" is some other map from this sequence: each map in the sequence has two identical graphical objects, namely graphical objects with texts "Tacit knowledge" and "Explicit knowledge".

6. Ordering in knowledge maps

Arranging the elements of a discrete set belongs to the tasks of basic metrics. In order to make a decision as to whether it is possible to arrange the elements of a given set, we have to know the following: a) a criterion by which we will arrange the elements, and b) the rule which will determine with the use of this criterion for each pair of elements: which element of the pair will be first and which will follow.

6.1. Definition

Let K be a knowledge map and $G = \{g_1, g_2, \ldots, g_j\}$, i = 1, 2, …, m be its Graphics G. Let on the all the elements of graphics G there is defined the property $P(g_i)$, which enables the comparison of the elements according to the given criterion:

1) We will say that the element g_s follows the element g_r if and only if $P(g_r) \prec P(g_s)$ is valid. We will then call the element g_s a *successor* of the element g_r, and the element g_r the *predecessor* of the element g_s.
2) If $P(g_r) = P(g_s)$ is valid, we will say that the elements g_r and g_s are equivalent according to the criterion. In such a case it will be valid for the element g_r that it is a successor as well as a predecessor of g_s and, similarly, the element g_s is both the successor and predecessor of the element g_r.
3) The knowledge map will be *ordered* if $P(g_r) \leq P(g_s)$ is valid for its Graphics G. The knowledge map will be *well ordered* if $P(g_r) \prec P(g_s)$ is valid for its Graphics G.

6.2. Notes to definition

1. The terms "ordered" or "well ordered" are used in the theory of numbers, e.g. while ordering in the set of natural numbers we observe the property of the size of the number, and the criterion for comparison is the size of the number. The set of natural numbers is then a well ordered set. Similarly, we can study the ordering of elements in other sets of numbers.
2. The Graphics of a knowledge map is a discrete set containing objects which are not numbers. When we number these objects, we convert the process of ordering of geometric objects into the process of ordering of numbers. Numbers can be attached to objects even in some other manner, e.g. by using real scales (weights) and then we can seek an arrangement of the objects by the scales. In such cases the criterion for the ordering is the size of the real number.

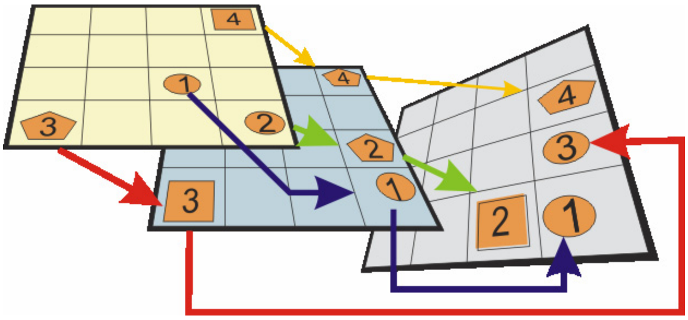

Fig. 2. Symbolic example of the layout of the three knowledge maps

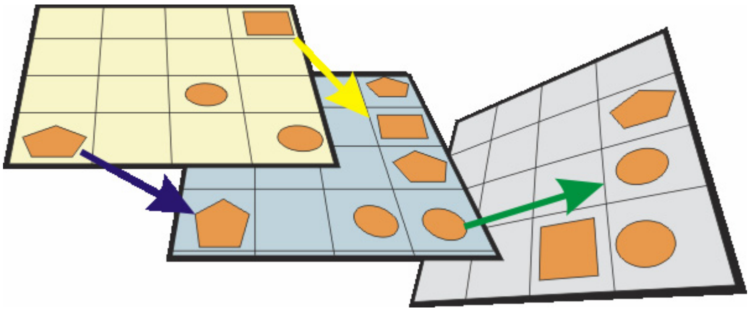

Fig. 3. Symbolic example of the series of the three knowledge maps

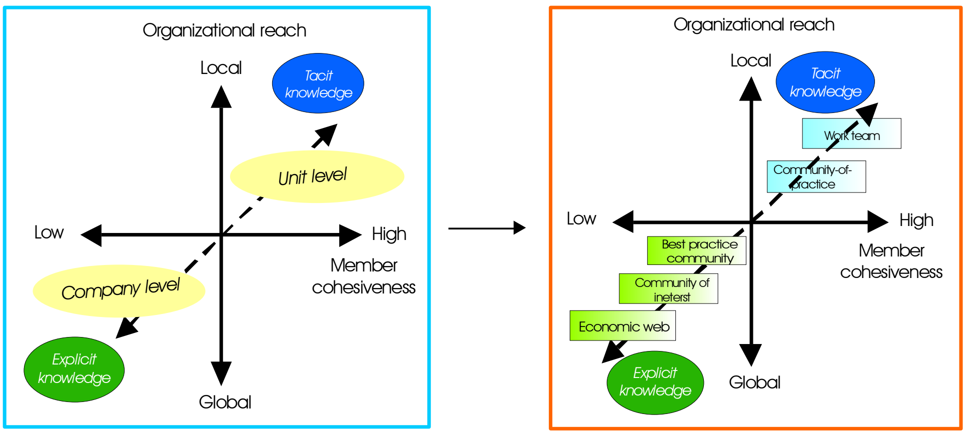

Fig. 4. Example of a sequence of two knowledge maps from a series

3. However, we can also observe other properties of objects, such as their shape, size, colour, or distance. Even here we can determine clear criteria for their arrangement, namely:
   – Shape of the object: triangle, square, n-angle; objects drawn in a dashed line, full line, double line, … .
   – Size of the object: volumes of images from the smallest to the largest, regardless of their shapes.
   – Shape and size of the object: combination of the previous two criteria.
   – Colour of the objects: sequence of the objects according to the richness of the colour, or according to the order shown on the colour table.
   – Distance of the object from other objects: a complete intersection, partial intersection, contact, distance of the gravity centre of the object.
4. In the Definition 6.1. it is not required for the Graphics to be formalised. The ordering of the graphical objects of the map is, therefore, not dependent on the formalisation, but only on the criterion and the rules for the ordering.
5. In a well ordered knowledge map there are no equivalent graphical objects g_{ij} .
6. From the Definitions 4.1. and 6.1. it can be deduced that:
   a) the bijective mapping holds the both ordering and the well ordering property of Graphics in all maps of the layer L(Ki);

b) the mapping in series $S(K_i)$ doesn´t hold the ordering property of the Graphics.

7. Well ordering of graphical objects in the knowledge map is an important characteristic, because it explains by what type of algorithm the problem was solved. Well ordered Graphics is often an indication that the solution of the relevant problem may be of type "step by step".

CONCLUSION

Knowledge map is a particular type of mathematical model. Standard mathematical models model and describe the properties of a system by means of standard mathematical means such are *analytical tools* (functions, equations, inequalities), *graphical tools* like graphs and diagrams and also by the *artificial intelligence* by simulations using computers. Knowledge map is based mainly on graphical environment but on the contrary with those mathematical tools it involves also very irregular and free graphical objects and very free unrestrained graphical structures. Thus, knowledge maps vary among broad continuum from the robust exact strong mathematical models to the soft free sketches and images. This freedom makes possible to use knowledge maps in many distinct descriptions of problems and their relevant solutions.

The ideas of the article incites challenge of seeking fundamental insights how to produce and present this new approaches to knowledge maps in order to help schools, organizations and people to nurture, harvest and manage the immense potential of knowledge, to help them to create new maps and measures and reinvent themselves in order to innovate and excel in the context of the knowledge society.

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Received for publication on November 14, 2007

Accepted for publication on March 10, 2008

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Metrika ve znalostních mapách.

Scientia Agric. Bohem., *39*, 2008: 111–116.

V článku se předkládá a) formulace definice pojmu znalostní mapa, b) zavedení metriky, která umožní klasifikaci a třídění znalostních map. Metodika spočívá ve formulaci definice znalostní mapy a odpovídajících pojmů a termínů, v analýze společných vlastností, které vykazují rozdílné znalostní mapy prezentované v odborné literatuře, ve výběru mír pro metriku, která umožní měření vlastností znalostní mapy.

Definice znalostní mapy: Znalostní mapa K se definuje jako sjednocení tří objektů $K = \cup\{S, G, T\}$, kde nosič S je nějaká omezená nebo neomezená oblast roviny nebo prostoru, grafika G je množina rovinných nebo prostorových geometrických objektů a text T je množina alfanumerických znaků. *Formalizace znalostní mapy:* Nosič je formalizovaný, jestliže je na něm vymezen souřadnicový systém s jednotkou a orientací. Grafika je formalizovaná, jestliže sestává z prvků teorie grafu. Znalostní mapa může být plně formalizovaná, částečně formalizovaná nebo neformalizovaná. *Zobrazení mezi prvky Grafiky v množině znalostních map:* Mezi grafickými objekty znalostních map se definují dva typy zobrazení: a) bijektivní zobrazení na množinu a b) zobrazení do množiny nebo na množinu. *Vrstva a sled znalostních map:* Jestliže existuje jedno jednoznačné bijektivní zobrazení mezi prvky $g_{ij} \in G_i$, říkáme, že znalostní mapy tvoří *vrstvu* $L(K_i)$ znalostních map. Jestliže existuje nějaké zobrazení mezi každou dvojicí grafik $\{g_{i-1,j}, g_{ij}\} \in \{G_{i-1}\ G_i\}$, znalostní mapy tvoří *sled* $S(K_i)$ znalostních map. *Uspořádání ve znalostních mapách:* Grafické prvky znalostní mapy lze uspořádat, pokud jsou k dispozici a) kritérium uspořádání, b) pravidlo, které podle tohoto kritéria pro každou dvojici prvků stanoví, který z prvků dvojice následuje a který předchází. Mapa K je *uspořádaná*, jestliže pro její grafiku G platí $P(g_r) \leq P(g_s)$. Znalostní mapa je *dobře uspořádaná*, jestliže pro její grafiku platí $P(g_r) \prec P(g_s)$.

znalostní mapa; nosič znalostní mapy; grafika; vrstva znalostních map; sled znalostních map

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