MATHEMATICAL DESCRIPTION OF RAPE SEEDS' (*BRASSICA NAPUS* L.) MIXTURE MECHANICAL BEHAVIOR UNDER COMPRESSION LOADING

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This article is concentrated on the utilization of tangent curve equation for description of mechanical behavior of rapeseeds (*Brassica napus* L.) pressed mixture under compression loading. This experiment was aimed at determining of the general equation describing deformation characteristics of rapeseeds pressed mixtures under compression loading and at verifying of this equation. Pressing vessel with diameter 76 mm was used where different pressing heights were performed for the rapeseeds pressed mixtures. The results from the experiment showed that mechanical behavior of the rapeseeds pressed mixtures under compression loading can be described by the general tangent curve function. From the determined coefficients of determination and also statistical analysis it was shown that fitted tangent curve function described the measured amounts exactly for different pressing heights of the rapeseeds. The determined general equation can be used for description of mechanical behavior of rapeseeds mixture under compression loading.

deformation characteristics; mechanical, physical properties; pressing; tangent curve

INTRODUCTION

In literature, there are available information on physical properties of rapeseeds (Calisir et al., 2005; Izli et al., 2009), but knowledge on mechanical behavior of rapeseeds mixtures under compression loading is very limited (Dovzhenko, 2001; Sukumaran, Singh, 1989; Rusinek et al., 2007; Stasiak et al., 2007; Unal et al., 2009; Tys, Szwed, 2000). Therefore detailed description of the shape of the deformation characteristic of rapeseeds under compression loading is necessary for precisely determining the magnitude of the compressive force, strain energy and modulus of elasticity in compression (Očenášek, Voldřich, 2009). The shape of these curves and its distribution in deformation diagram are the most important factors needed for accurate determination of pressing energy performance (Fomin, 1978). From previously experiments with pressing of oilseed crops and annual plant seeds (Herák et al., 2007, 2010) it was known that for describing mechanical behavior of pressed mixture under compression loading, tangent curve equation (Eq. 1) can be used (Herák, 2009), where F (N) is compressive force, A (N) is force coefficient of mechanical behavior, B (mm^{-1}) is deformation coefficient of mechanical behavior and x (mm) is deformation.

$$F = A \cdot \tan(B \cdot x) \tag{1}$$

This equation is valid only for linear pressing where rigidity of pressing vessel is much greater than rigidity of pressing mixture and deformations in a direction perpendicular to line of action of compressive force is limited.

The aim of this experiment was to determine general equation which can be described by the mechanical behavior of rapeseeds (*Brassica napus* L.) under compression loading and to verify this equation for different pressing heights of the rapeseeds using pressing vessel with diameter 76 mm.

MATERIAL AND METHODS

Sample

In this experiment, cleaned rapeseeds (*Brassica napus* L.) obtained from Czech Republic was used. Three experimental samples for each compressed mixtures were prepared from these seeds and their physical properties are presented in Table 1. The moisture content Mc (% d.b.) of the pressed mixtures was determined using the ASTA 1985 method (Olaniyan, Oje, 2002) with a temperature setting of 105 °C and a drying time of 17 h, the weight of the mixtures m_s (g) was determined using equipment Kern 440–35-N and the porosity P_f (%) was calculated from bulk and true density using the relationship given by porosity formula (Eq. 2) (Blahovec, 2008).

$$P_f = \left(1 - \frac{\rho_b}{\rho_t}\right) \cdot 100 \tag{2}$$

Where: P_f (%) is the porosity, ρ_b (kg m^{-3}) is the bulk density and it was determined as the weight of the sample divided by initial volume of pressing vessel V (m^3) and ρ_t = 1080 kg m^{-3} is the true density and it was determined for average moisture content of each sample from the previously experiments (Izli et al., 2009).

Table 1. Determined physical properties of rapeseeds pressed mixture, data in the table are means ± SD

H (mm)	*V* (10^5 . mm^3)	*m* (g)	*Mc* (% d.b.)	*CV* (%)	P_f (%)
20	0.907 ± 0.037	48.1 ± 0.9	7.1 ± 0.1	5.12	43.01 ± 0.36
30	1.361 ± 0.045	82.3 ± 0.7	6.9 ± 0.2	6.32	44.21 ± 0.52
40	1.815 ± 0.047	108.8 ± 1.8	7.2 ± 0.1	7.45	44.89 ± 0.48
50	2.268 ± 0.039	144.2 ± 1.7	7.1 ± 0.2	6.95	41.22 ± 0.62
60	2.722 ± 0.046	166.5 ± 1.2	6.8 ± 0.2	5.54	43.53 ± 0.41
70	3.176 ± 0.041	202.4 ± 1.9	7.2 ± 0.2	7.26	41.10 ± 0.53
80	3.629 ± 0.038	224.6 ± 2.1	6.9 ± 0.1	6.53	42.85 ± 0.49

H – different pressing height of rapeseeds pressed mixture, *V* – initial volume of pressed mixture, *m* – mass of pressed mixture, *Mc* – moisture content of the pressed mixture in dry basis, *CV* – coefficient of variation of compression force, P_f – porosity of pressed mixture

Compression test

To determine the development of the magnitude of the pressing force depending on the linear deformation, the ZDM 50 – 2313/56/18 pressing device was used to record the course of deformation function in an exact and analogical manner. Other devices including a pressing plunger and pressing vessel with diameter *D* = 76 mm (Fig. 1) were also used. The mixture of samples, that is individual samples plus air space, were placed into the pressing vessel and height of the mixture layers was measured to be *H* = 80 mm, 70 mm, 60 mm, 50 mm, 40 mm, 30 mm, 20 mm, respectively. The mixtures were pressed under the temperature of 20 ºC and the pressing rate was 1 mm s^{-1}. The experiment was repeated for each pressed mixture three times. The measuring range of force was between 0N and 100 kN, in which the test was stopped. Individual points of measurement were digitally recorded and analyzed with each new addition of deformation of 0.5 mm.

Determination of general curve

The measured amounts of compressive force and deformation of different pressing heights of rapeseeds were analyzed with computer program Mathcad 14 (Pritchard, 1998), which uses Levenberg-Marquardt algorithm for data fitting (Marquardt, 1963). Tangential function (Eq. 1) was also used in this experiment. General curve for this experiment was simply determined by mathematical adjustment of this tangential function. To multiply the value in the brackets of the tangential function (Eq. 1) by ratio *H*/*H*, this equation in modified formula (Eq. 3) can be obtained, where *H* (mm) is different height of rapeseeds pressed mixture.

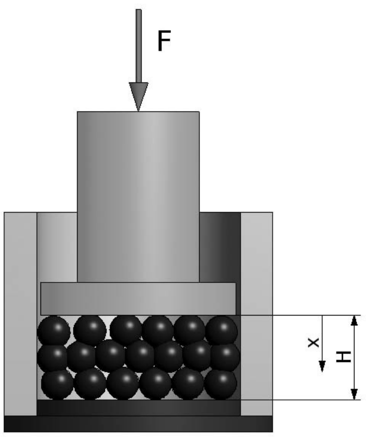

Fig. 1. Scheme of pressing equipment

$$F(x) = A \cdot \tan(B \cdot \frac{H}{H} \cdot x) \quad (3)$$

The product of the deformation coefficient of mechanical behavior *B* and different pressing height of pressed mixture *H* can also be expressed by equation (Eq. 4), where *G* (–) is a compression coefficient.

$$G = B \,.\, H \quad (4)$$

The assumption was that the dependency between force coefficient of mechanical behavior *B* and different pressing height of rapeseeds pressed mixture *H* could be the dependency, which can be described by the hyperbolic function (Eq. 5).

$$B(H) = \frac{G}{H} \quad (5)$$

By substituting equation (Eq. 4) into equation (Eq. 3), general curve (Eq. 6) can be obtained. The dependency by this general equation (Eq. 6) between compressive force and deformation of pressed mixture for different pressing heights of rapeseed is valid only for diameter of pressing vessel 76 mm.

$$F(x) = A \cdot \tan(G \cdot \frac{x}{H}) \quad (6)$$

RESULTS AND DISCUSSION

The average measured amounts of different pressing heights of rapeseed pressed mixture are presented in Fig. 2 and fitted curve are also described. For fitting of measured amounts of tangent curve equation (Eq. 1) and its determined coefficients are shown in Table 2. Statistical analysis ANOVA calculated using MathCAD 14 software for level of significance 0.05, shows that the values of F_{crit} were higher than F_{ratio} values for all measured pressed mixtures and amounts of P_{value} were higher than significance level 0.05. This shows that tangent curve equation (Eq. 1) can be used for fitting measured amounts since

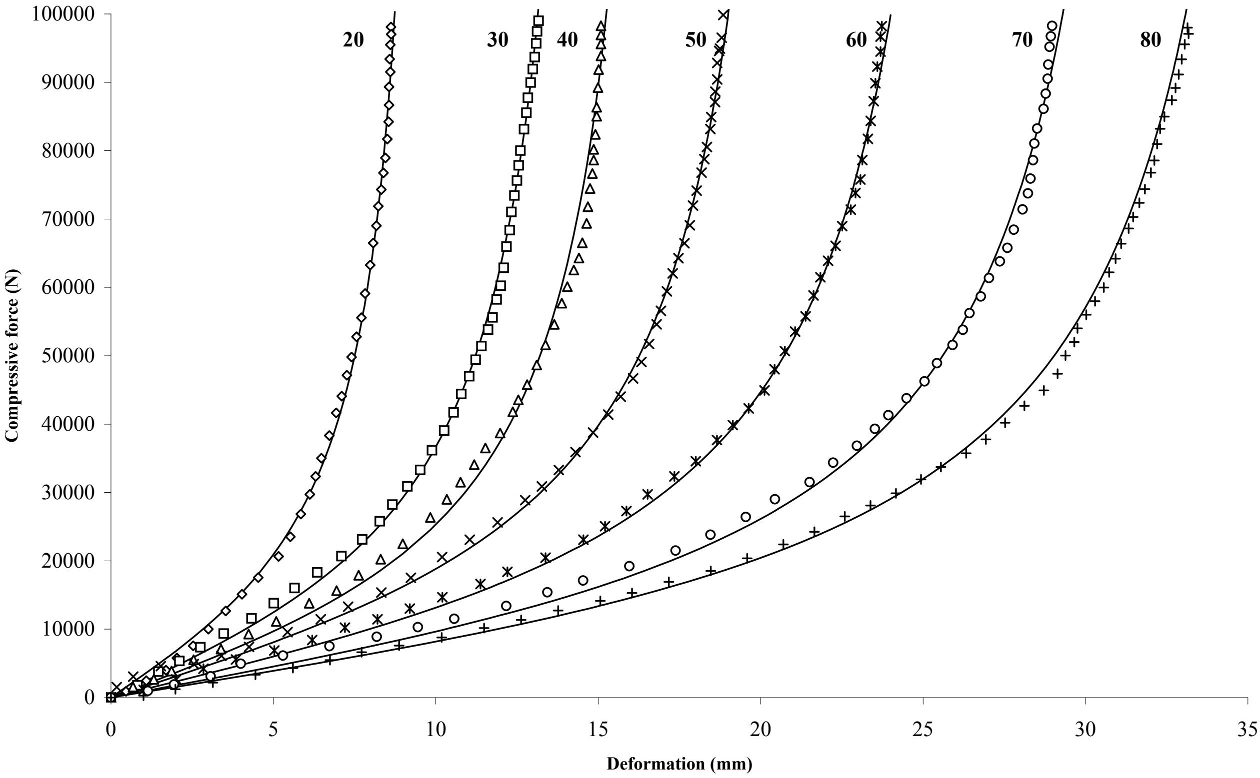

Fig. 2. Measured amounts of mechanical characteristic of rapeseeds pressed mixtures and their fitted functions

Table 2. Determined coefficients of deformation characteristics for rapeseeds pressed mixtures and their statistical analysis

H (mm)	A (N)	B (mm^{-1})	F_{ratio} (–)	F_{crit} (–)	P_{value} (–)	R^2 (–)
20	$2.116.10^4$	0.156	$3.501.10^{-4}$	3.963	0.913	0.996
30	$2.203.10^4$	0.103	$7.206.10^{-4}$	3.952	0.911	0.999
40	$2.009.10^4$	0.090	$6.534.10^{-3}$	4.949	0.909	0.989
50	$2.172.10^4$	0.071	$7.422.10^{-3}$	3.953	0.916	0.998
60	$2.053.10^4$	0.057	$2.297.10^{-3}$	3.949	0.911	0.998
70	$1.891.10^4$	0.047	$5.467.10^{-3}$	3.945	0.903	0.995
80	$1.833.10^4$	0.042	$2.72.10^{-4}$	3.927	0.902	0.999

H – different pressing height of rapeseeds pressed mixture, A – force coefficient of mechanical behavior, B – deformation coefficient of mechanical behavior, F_{ratio} – value of the F-test, F_{crit} – critical value that compares a pair of models, P_{value} – the significance level at which it can be rejected the hypothesis of equality of models, R^2 – coefficient of determination

relationships between measured amounts and tangent curve amounts were statistically significant. All values of F_{crit}, F_{ratio} and P_{value} are presented in Table 2. Also the coefficients of determination R^2 were highly significant and this shows that fitted curves describe accurately deformation characteristics of rapeseed for all investigated heights of pressed mixtures. For each individual experiment, three trials for each pressing height, it was determined coefficient of variation of compression force (Table 1), the means of these amounts are $CV = (6.45 \pm 0.87)$ %. It is also evident by theoretical assumption that the force coefficients of mechanical behavior A (Table 2) for different pressing heights of pressed mixture can be substituted by average amount, the means is $A = (2.039 \pm 0.139).10^4$ N and estimate absolute uncertainties at significant level 0.05 is $AU_A = 0.124.\ 10^4$ N, these coefficients and pressing heights of pressed mixture are presented in Fig. 3. From the theoretical assumption (Eq. 4) and by graphical displaying of deformation coefficients of mechanical behavior B and different heights of pressed mixtures it shows clearly that there are dependencies between these amounts (Fig. 4). These amounts were fitted by hyperbolic function (Eq. 5) which is numerically described by Eq. 6 and it is evident that compression coefficient was determined as $G = 3.347 \pm 0.196$ with the estimate absolute uncertainties at significant level 0.05 $AU_G = 0.175$.

$$B(H) = \frac{3.347}{H} \qquad (7)$$

This fitted (Eq. 7) was also statistically analyzed by ANOVA with significance level 0.05 and from calculated amounts presented in Table 3 it is evident that fitted hy-

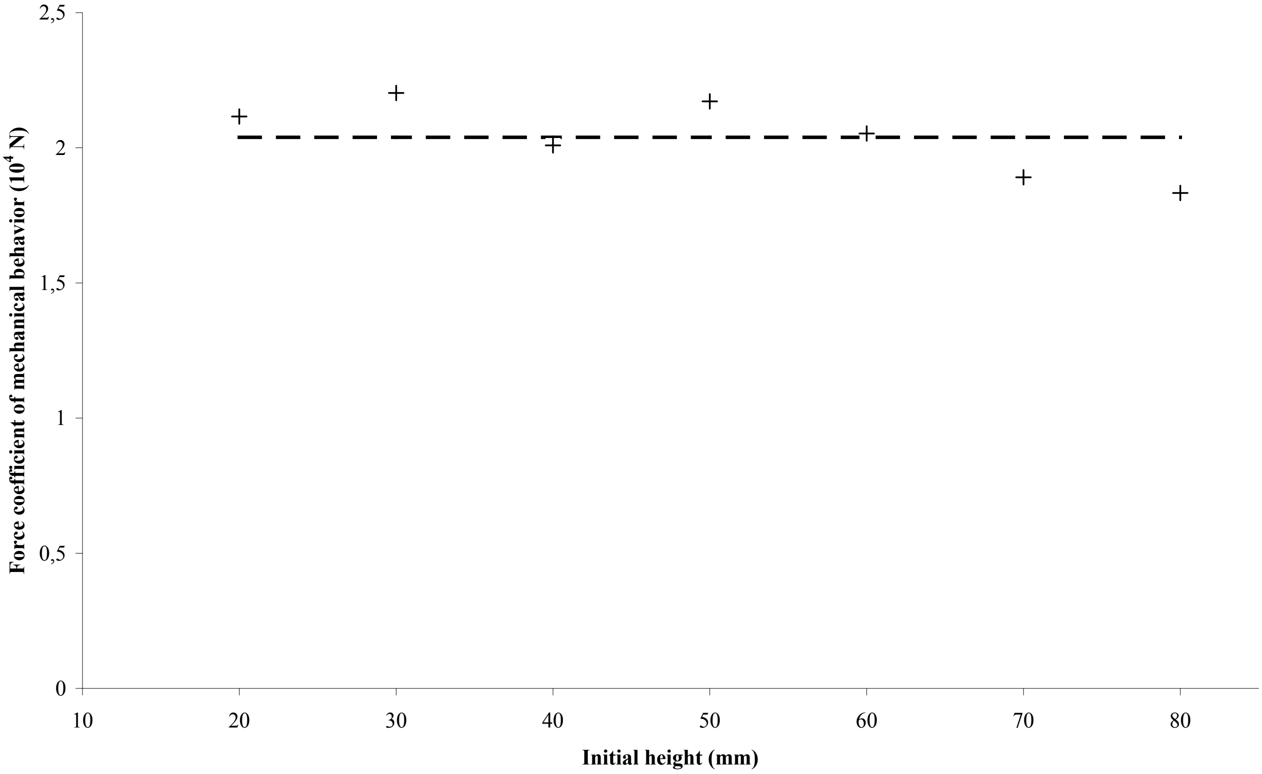

Fig. 3. Dependency between force coefficients of mechanical behavior and initial pressing height

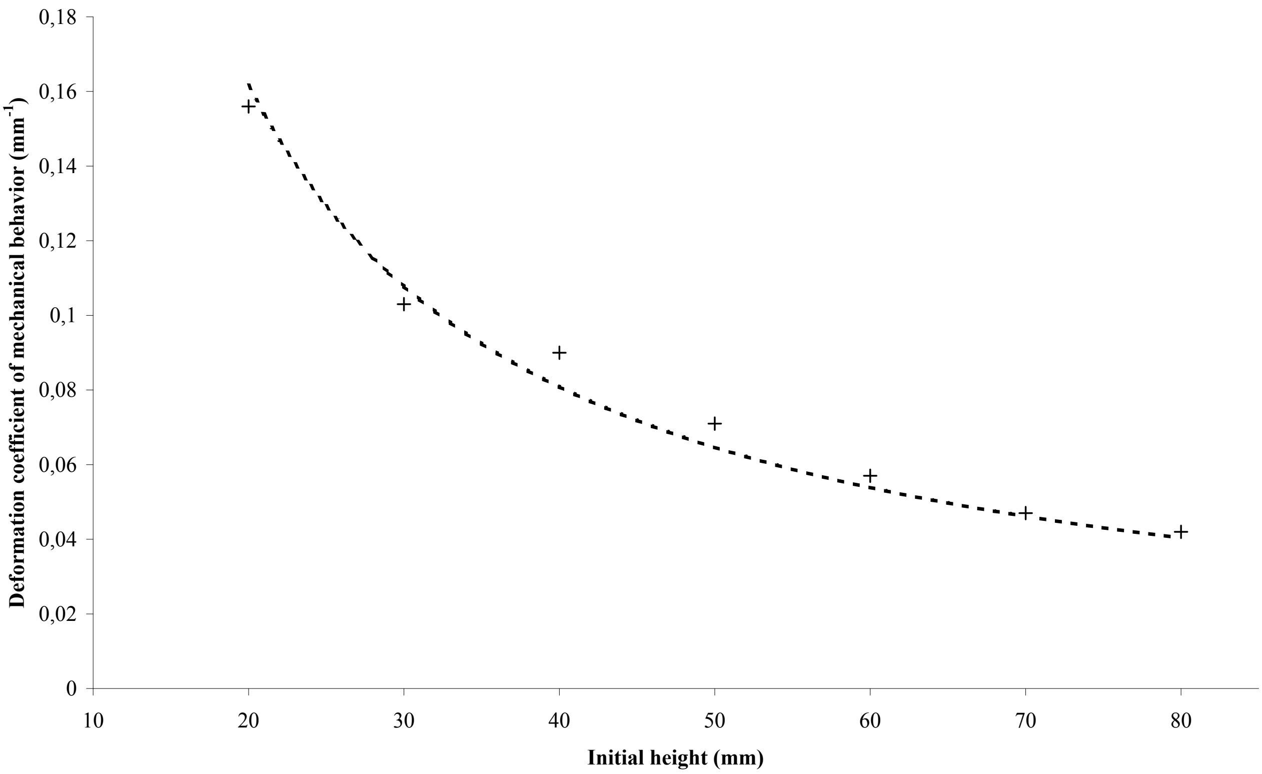

Fig. 4. Dependency between deformation coefficients of mechanical behavior and initial pressing height

Table 3. Amounts of statistical analysis of the dependency between compression coefficient and different pressing height of rapeseeds pressed mixture

Equation	F_{ratio} (–)	F_{crit} (–)	P_{value} (–)	R^2 (–)
Eq. 7	$4.741.10^{-3}$	4.747	0.994	0.998

F_{ratio} – value of the F-test, F_{crit} – critical value that compares a pair of models, P_{value} – the significance level at which it can be rejected the hypothesis of equality of models, R^2 – coefficient of determination

perbolic curve (Eq. 7) can be used for describing dependency between the deformation coefficient of mechanical behavior and different heights of pressed mixtures amounts. Accuracy of fitting function is also evident from the coefficients of determination which are also close to one. From table of physical properties (Table 1) shows that moisture content and porosity can be also substituted by average amounts as means of porosity is P_f = (42.97 ± 0.18) % and means of moisture content in dry basis is Mc = (7.0 ± 0.1) % (d.b.). This explains that porosity and moisture content are constant for all conducted experiments in this study. Substituting calculated average amounts of force coefficient of mechanical behavior and compression coefficient in equation (Eq. 6) it can be determined general equation (Eq. 8).

$$F = 2.039 \cdot 10^4 \cdot \tan\left(3.347 \cdot \frac{x}{H}\right) \quad (8)$$

This general equation (Eq. 8) was statistically tested by ANOVA with measured deformation characteristics for different pressing heights and the results from this analysis at significant level 0.05 are presented in Table 4. From the results of the statistical analyses it is evident that the general curve (Eq. 8) is statistically significant and it can be used for description of mechanical behavior of rapeseeds mixture under compression loading for diameter of pressing vessel D = 76 mm. The results as well as previously experiments carried out shows that the mechanical characteristic determined with aid of fitted amount by tangent equation are similar to the results reported by other authors using different methods. That is the classic method for determining the mechanical characteristics of plant crops based on the basis of Darcy's law applied by Fomin (1978) and determined theory are similar to the amount from experiments described in this manuscript. Further, some authors established in their research on the experimental results of Fomin (1978) and applied numerical methods to the Fomin's theory (Očenášek, Voldřich, 2009). In addition, some experiments were carried out at different compression systems such as unaxial compression apparatus with cuboid chamber (Rusinek et al., 2007) where the oil point and the dependency between compression force and deformation were determined (Sukumaran, Singh, 1989). Using the fundamental equations of 3D stress analysis and spatial combination with the friction it can also be described deformation characteristics of oil bearing crops (Herák et al., 2007). It is also clear from the experiments with different oil bearing crops (Herák, 2009; Herák et al., 2010) that this theory can be used for determining mechanical behavior of oil bearing crops mixtures under compression loading. It is also evident that the shape and distribution of deformation characteristics depend on several factors such as physical properties, mechanical properties and process of the pressing (Herák et al., 2010; Fomin, 1978). The friction between seeds and pressing vessel and seeds also has effect on the dependency between the compression force and deformation (Dovzhenko, 2001; Očenášek, Voldřich, 2009; Herák, 2009; Fomin, 1978). The advantage of using tangential curve is that it is unnecessary to resolve individual particles and their properties and relationships between particles but this method use the mixtures of the seed as a unit that is affected by constrains between the pressing vessel and seed's mixture and the process of the pressing.

CONCLUSION

The general equation (Eq. 8) of mechanical behavior of rapeseeds (*Brassica napus* L.) under compression loading was derived in this study and it can be used for description of mechanical behavior of rapeseeds under compression loading for diameter of pressing vessel D = 76 mm. The coefficients of this equation were also determined as force coefficient of mechanical behavior A = (2.039 ± 0.139).10^4 N and compression coefficient G = 3.347 ± 0.196. This general curve was statistically analyzed and it is evident that the amounts of the measured deformation curve and the amounts determined from the general equa-

Table 4. Amounts of statistical analysis of general equation (Eq. 8) and measured deformation characteristics of different pressing height of rapeseeds

H (mm)	F_{ratio} (–)	F_{crit} (–)	P_{value} (–)	R^2 (–)
20	0.132	4.494	0.721	0.937
30	0.151	4.260	0.701	0.987
40	0.514	4.149	0.479	0.967
50	0.710	4.085	0.404	0.981
60	0.077	4.043	0.782	0.968
70	0.260	4.013	0.612	0.969
80	0.164	3.991	0.687	0.970

H – different pressing height of rapeseeds pressed mixture, F_{ratio} – value of the F-test, F_{crit} – critical value that compares a pair of models, P_{value} – the significance level at which it can be rejected the hypothesis of equality of models, R^2 – coefficient of determination

tion (Eq. 8) are statistically significant. The porosity P_f = (42.97 ± 0.18) % and moisture content Mc = (7.0 ± 0.1) % (d.b.) of rapeseeds pressed mixture were determined as constant and these amounts have no influence on the different pressing heights of the rapeseeds pressed mixture.

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HERÁK, D. – KABUTEY, A. – SEDLÁČEK, A. (Česká zemědělská univerzita, Technická fakulta, Praha, Česká republika):

Matematický popis mechanického chování směsi semen řepky olejné (*Brassica napus* L.) pod tlakovým zatížením.

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Tento článek je zaměřen na využití rovnice tangentoidy pro popis mechanického chování směsi semen řepky olejné (*Brassica napus* L.) pod tlakovým zatížením. Cílem tohoto experimentu bylo stanovit obecnou rovnici popisující deformační vlastnosti směsi semen řepky olejné pod tlakovým zatížením a tuto rovnici ověřit. Pro experiment byla použita lisovací nádoba o průměru 76 mm, ve které byla lisovaná směs řepky olejné stlačována při různých počátečních výškách lisování. Výsledky experimentu prokázaly, že mechanické chování směsi semen řepky olejné pod tlakovým zatížením může být popsáno obecnou tangentoidní křivkou. Ze zjištěných koeficientů determinace a také statistické analýzy bylo též prokázáno, že naměřené hodnoty pro různé výšky směsi semen řepky olejné se shodují s hodnotami stanovenými tangentoidní křivkou. Stanovená obecná rovnice může být použita pro popis mechanického chování směsi semen řepky olejné pod tlakovým zatížením.

deformační charakteristika; mechanické, fyzikální vlastnosti; lisování; nádoba; tangentoida

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