MATHEMATICAL MODELS FOR DETERMINATION OF LOADING CAPACITY – CASE STUDY ON WOODEN SCARF BONDED JOINT WITH PU ADHESIVE*

O. Dajbych, D. Herák, A. Sedláček

Czech University of Life Sciences Prague, Faculty of Engineering, Department of Mechanical Engineering, Prague, Czech Republic

This paper is focused on the suitability of estimation of theoretical methods of wooden scarf joint bonded with PU adhesive loading capacity determination. Loading capacity of selected joint was experimentally determined for several wooden materials and bevel angles from 0° to 90°. Theoretical dependencies by using two different methods for selected joints were calculated from values for 0° and 90° and results compared. The first method was based on tangential on normal stress dependency substitution by ellipse. The second one was based on joint surface virtual division into 2 sections. Normal shear stress was applied on the first and the second sections. Results of the experiment showed that selected methods can be used for joints from 0° to approx. 70° of bevel angle with acceptable accuracy.

wood; scarf joint; bonding; loading capacity; PU adhesive

INTRODUCTION

Wooden bonded joint design is based on experience followed by experimental testing of product prototypes. This process increases time demand and development costs. On the other hand, the methods based on intermolecular bond characteristics give very low accurate results (usually 50% and less) and they are absolutely inapplicable for practical exploitation (Peterka, 1980; Ozcifci, 2007).

The previously published method (Herák et al., 2009) of wooden bonded scarf joint loading capacity determination requires measurement of the whole stress on bevel angle dependency for given wood and adhesive combinations. This article focuses on the previously published methods (Dajbych et al., 2010; Dajbych, 2010), which requires only knowledge of ultimate force for clearly normal and tangential strain. The first method is based on tangential on normal stress dependency substitution by ellipse. The second method brings another simplification, as its result is not in contrast to the previous method irrational function by maintaining the same input requirements. The second one was based on joint surface virtual division into two sections. Normal stress and shear stress were applied on the first and the second sections.

MATERIALS AND METHODS

Sample

Typical wood materials, which are used in constructions and e.g. in furniture industry, were taken as a basic material for experiments. Concretely speaking: spruce (*Picea abies*), pine (*Pinus silvestris*), larch (*Larix decidua*), basswood (*Tilia cordata*), oak (*Quercus robur*) woods. Wood moisture content was 8–12% depending on wood material type and experiment period, in view of the fact that experiments proceeded within several months, when natural process of drying was in progress. Moisture content was determined according to the Czech standard (ČSN EN 13183-2, 2002) using electric moisture meters for wood TFA 30.5502, which uses the resistive principle. There was no significant effect of wood moisture content variation in given range on glued joint parameters.

Basic elements for samples were wooden blocks with the following dimensions: 15 mm × 20 mm ± 1 mm, thus 300 mm^2 cross section, with length of 200 mm ± 10 mm. These blocks were cut with saw under bevel angle of 0°, 15°, 30°, 45°, 60°, 75° and 90°. Then the surfaces of cuts were modified on angle cutter to get exact angles, purified, degreased and prepared by adhesive producer instructions and according to the Czech standard (ČSN EN 205, 2003). Special equipment for set of 21 samples was designed to keep necessary time and force for proper joints hardening.

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Fig. 1. Set of samples prepared for one experiment period

These 21 samples were prepared for each experiment period, thus 3 for each angel (Fig. 1.).

UHU PU Max glue was used, which is universal PUR (polyurethane) construction adhesive. This type of adhesive is used for joints with high load, it is water resistant - class D4 - according to Č S N E N 2 0 4 and resistant to temperatures from –20°C to +100°C. Basic information about the given adhesive can be found in its technical sheet.

Experiment

Samples were brought to failure on shredder UTS 50 Testsysteme after bonding process completion. Speed of shredding process was 0.05 mm/s. Some samples, especially those with smaller bevel angles, could be used again after affected material layer removal and surface re-preparation. Other samples had to be replaced with new ones due to vaster damage. The goal was to gain 10 valid values of force necessary for sample failure for each bevel angle. The experiment was considered as invalid when the value significantly missed the set of other values or the breach took place out of joint or in its small part (Č S N E N 2 0 5 , 2 0 0 3). It is evident that several hundred of single experiments had to be carried out.

Theoretical method derivation

Requirements for both the methods were to preserve the need for only basic bond characteristics, thus tangential and normal bond strengths represented by ultimate force for bevel angle 0° and 90°. These basic values were obtained during experiments.

The first method (D a j b y c h e t a l . , 2 0 1 0) is based on tangential on normal stress dependency substitution by ellipse. After substitution and simplification, the following equation was obtained (1)

$$F = \frac{\sqrt{F_0{}^2 \cdot \cos^2 \alpha + F_{90}{}^2 \cdot \sin^2 \alpha}}{\cos \alpha} \qquad (1)$$

where:

F is theoretical force necessary to bring the sample with random bevel angle to failure,

F_0 is force necessary to bring the sample with bevel angle of 0° to failure,

F_{90} is force to bring the sample with bevel angle of 90° to failure and α is bevel angle.

The second method (D a j b y c h , 2 0 1 0) is based on joint surface virtual division into two sections. Normal stress and shear stress were applied on the first and the second sections. Results described (2).

$$F = \frac{F_0 + F_{90} \cdot \mathrm{tg}\, \alpha}{\sin \alpha + \cos \alpha} \qquad (2)$$

where:

F is theoretical force necessary to bring the sample with random bevel angle to failure,

F_0 is force necessary to bring the sample with bevel angle of 0° to failure,

F_{90} is force to bring the sample with bevel angle of 90° to failure and α is bevel angle.

Table 1. Experimentally determined ultimate force F in N

Material/α	0°	15°	30°	45°	60°	75°	90°
Spruce	1615 ± 179	2448 ± 301	2464 ± 451	2981 ± 380	4314 ± 628	7321 ± 974	2787 ± 350
Pine	1861 ± 327	2419 ± 316	2548 ± 421	3442 ± 586	5137 ± 548	6465 ± 854	2561 ± 232
Larch	1412 ± 221	1624 ± 174	2131 ± 251	2607 ± 484	4655 ± 612	7871 ± 870	2558 ± 562
Oak	2061 ± 264	2066 ± 235	2778 ± 440	3034 ± 419	4754 ± 636	7291 ± 789	2736 ± 461
Basswood	2168 ± 339	2425 ± 256	2374 ± 347	2883 ± 302	3746 ± 481	7205 ± 906	2310 ± 225

α – bevel angle. Data in the table are means ± SD

Table 2 Theoretical ultimate force F in N determined by (1)

Material/α	0°	15°	30°	45°	60°	75°	90°
Spruce	1615	1780	2280	3222	5091	10527	–
Pine	1861	1984	2377	3166	4810	9737	–
Larch	1412	1569	2043	2922	4651	9651	–
Oak	2061	2187	2597	3425	5168	10417	–
Basswood	2168	2255	2545	3168	4550	8889	–

α – bevel angle

Table 3. Theoretical ultimate force F in N determined by (2)

Material/α	0°	15°	30°	45°	60°	75°	90°
Spruce	1615	1929	2361	3113	4717	9813	–
Pine	1861	2080	2445	3127	4609	9323	–
Larch	1412	1712	2115	2807	4277	8948	–
Oak	2061	2281	2665	3392	4978	10020	–
Basswood	2168	2276	2563	3166	4516	8809	–

α – bevel angle

Table 4. Statistical analysis of given methods and measured values dependency

Material	Method	F_{ratio} (–)	P_{value} (–)	F_{crit} (–)	R^2 (–)
Spruce	substitution by ellipse	0.1197	0.7365	4.9646	0.9851
	surface division	0.0699	0.7969	4.9646	0.9866
Pine	substitution by ellipse	0.0576	0.8432	4.9646	0.8951
	surface division	0.0367	0.8518	4.9646	0.8898
Larch	substitution by ellipse	0.0394	0.8467	4.9646	0.9878
	surface division	0.0111	0.9183	4.9646	0.9836
Oak	substitution by ellipse	0.1734	0.6859	4.9646	0.9760
	surface division	0.1461	0.7103	4.9646	0.9768
Basswood	substitution by ellipse	0.3340	0.7320	4.9646	0.9956
	surface division	0.1194	0.7369	4.9646	0.9959

F_{ratio} – value of the F test; F_{crit} – critical value that compares a pair of models; P_{value} – the significance level. at which it can be rejected the hypothesis of equality of models; R^2 – coefficient of determination

RESULTS AND DISCUSSION

For each bevel angle the arithmetic mean of ultimate force valid values was calculated. Results were determined in Table 1. Fitting of measured amounts equation (1) is in Table 2 and results for the second method (2) are shown in Table 3. It is evident that theoretical values for bevel angle 90° cannot be calculated because theoretical joint surface is infinite, so is the theoretical ultimate force.

Statistical analysis calculated using Microsoft Excel 2007 software, the method ANOVA for level of significance 0.05, shows that the values of F_{crit} were higher than F_{ratio} values for all measured samples and amounts of P_{value} were higher than significance level 0.05 for both the methods. This shows that both

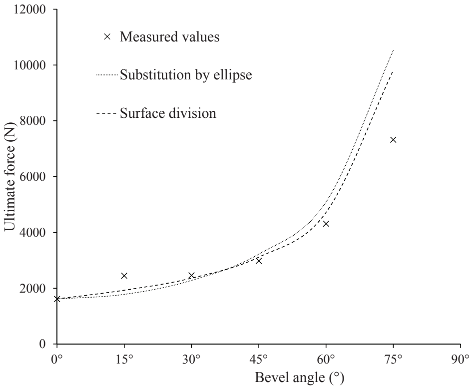

Fig. 2. Graphical representation of measured values and two theoretical dependencies for spruce wood

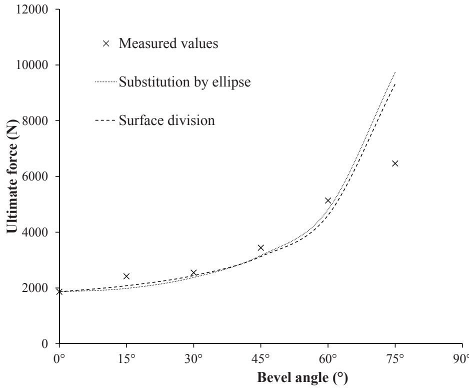

Fig. 3. Graphical representation of measured values and two theoretical dependencies for pine wood

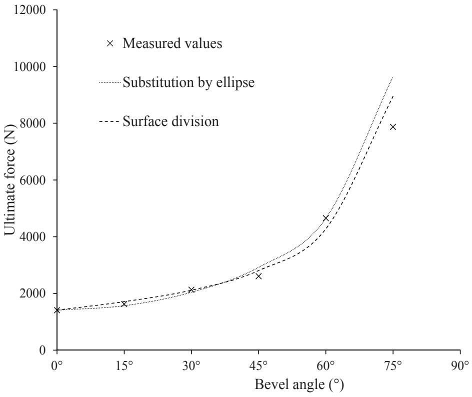

Fig. 4. Graphical representation of measured values and two theoretical dependencies for larch wood

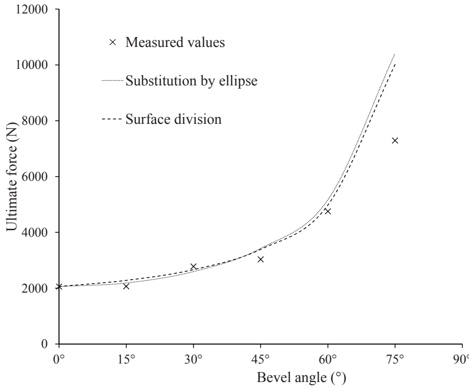

Fig. 5. Graphical representation of measured values and two theoretical dependencies for oak wood

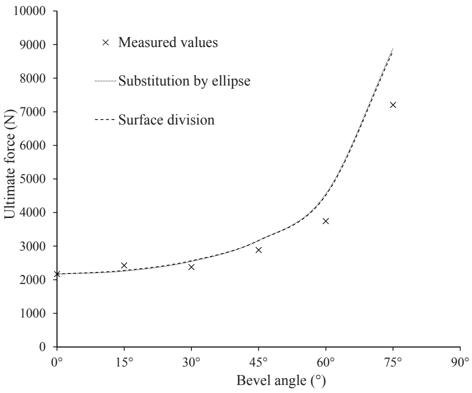

Fig. 6. Graphical representation of measured values and two theoretical dependencies for basswood

equations (1) and (2) can be used for fitting measured amounts since relationships between measured amounts and tangent curve amounts were statistically significant. All values of F_{crit}, F_{ratio} and P_{value} are presented in Table 4. Also the coefficients of determination R^2 were highly significant and this shows that fitted curves describe accurately strength characteristics of wooden bonded scarf joint for all investigated bevel angles.

It is obvious that both methods showed similar results, which can be clearly seen on graphs – (Fig. 2 to Fig. 6). Method using surface division appears to be more suitable which follows from statistical analysis (Table 4) and with regard to simpler formula (2). It is possible that for other adhesive and wood material combinations statistical results could be more positive for substitution by the ellipse method. However, with regard to wide result range (obvious by standard deviations in Table 1) mostly caused by anisotropy and heterogeneity of wood, the benefits of using simpler formula (2) would be higher than slightly more accurate but more complicated formula (1). This will have to be proved or displaced by future research.

The experiment confirmed that adhesive joints design with shear loading dominance are preferable unlike tensile loaded ones (Malyshev, Salganik, 1984; Ozcifci, 2007). It was also confirmed that theoretical and real strength of joint diverge from about 70° of bevel angle (Dajbych et al., 2010; Dajbych, 2010). It is most probably caused by high angle of bevel. Then the tips of bonded material are very sharp and thin, thus with regard to heterogeneity and anisotropy of wood, the load capacity of basic material in direction perpendicular to fibers multiple times is lower in contrast to parallel direction, what causes separation of thin tips of bonded parts, thus real active joint surface is smaller than theoretical one. Hence that the joints with bevel angle over 70° are not recommendable because space demands and adhesive consumption are increasing faster than real strength of joint, which means decreasing efficiency. It was confirmed as well that preparation and purification of surfaces and layers of material adjacent to joint is fundamental for bond quality (Oberk et al., 2000).

CONCLUSION

Results from the experiment showed that both the functions thus (1), (2) derived by selected methods are useful for theoretical determination of wooden bonded scarf joint loading capacity. Statistical analysis also proved that both functions are related to measured values with statistical significance. However, the second method and its mathematical representation (2) due to better statistical results and also formula simplicity is more recommendable to use. It is also recommended to add certain safety factor into calculations by this equation especially for higher bevel angle because theoretical functions diverge from real values in higher values direction (Fig. 2 to Fig. 6). Wooden material showed its typical characteristic (anisotropy and heterogeneity), which can be seen in Table 1 where standard deviation reaches almost 20% of mean value. This is naturally affected also by other factors as slightly different conditions of joint creation process due to long experiment duration and others. It was confirmed again that bevel angle, which exceeds approx. 70°, are not recommended due to decreasing efficiency of joint strength.

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DAJBYCH, O. – HERÁK, D. – SEDLÁČEK, A. (Česká zemědělská univerzita v Praze, Technická fakulta, Praha, Česká republika)

Matematické modely stanovení únosnosti – studie dřevěného spoje s úkosem lepeného polyuretanovým lepidlem

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Článek je zaměřen na stanovení vhodnosti použití teoretických metod pro stanovení únosnosti dřevěného lepeného spoje s úkosem. Skutečná únosnost daného spoje byla experimentálně stanovena pro různé dřevěné materiály a pro úhly úkosu od 0° do 90°. Poté byly s použitím dvou různých metod stanoveny teoretické závislosti únosnosti spoje na úhlu úkosu na základě hodnot pro 0° a 90° a výsledky porovnány. První metoda je založena na nahrazení závislosti tečného a normálového napětí ve spoji elipsou, druhá pak na myšleném rozdělení plochy spoje na dvě části, kdy první přenáší pouze normálové a druhá pouze tečné zatížení. Z experimentu vyplývá, že zvolené metody mohou být s dostatečnou přesností použité pro spoje s úhlem úkosu od 0° do cca 70°.

dřevo; spoj s úkosem; lepení; únosnost; polyuretanové lepidlo

Contact Address:

Ing. Oldřich D a j b y c h , Česká zemědělská univerzita v Praze, Technická fakulta, katedra mechaniky a strojnictví, Kamýcká 129, 165 21 Praha 6-Suchdol, Česká republika, tel: +420 224 383 186, email: dajbych@tf.czu.cz