

# COMPARISON OF TWO ALTERNATIVE OPTIMIZATION TECHNIQUES FOR SPATIAL HARVEST PLANNING\*

J. Kašpar, R. Marušák, P. Vopěnka

*Czech University of Life Sciences Prague, Faculty of Forestry and Wood Sciences, Prague, Czech Republic*

The timber harvest indicators developed during the 20<sup>th</sup> century have become ineffective and less useful in the Czech Republic due to social, political as well as environmental changes. More effective and efficient forest management can be achieved with the help of optimization methods that can identify the spatial location of a harvest area. However, just few studies have applied such optimization methods to the Czech Republic's forests. Therefore, it is important to apply different optimization methods and evaluate the results. In this paper we choose two different methods – integer programming and simulated annealing. We have created a simple mathematical model for the optimization of the final cut for three future periods, with each period lasting ten years. The potential harvest units for a real forest enterprise in the Czech Republic were analyzed using a geographical information system (ArcGIS). The neighbourhood relationships between the units were analyzed and spatial constraints were created for the model by using an adjacency matrix. The applications of both methods of optimization in forest harvest planning for a real forest management area in the Czech Republic are compared. The results of the calculations and the comparisons with current timber harvesting indicators are also presented.

integer programming; simulated annealing; spatial constraints; GIS

## INTRODUCTION

Due to expanding human activity, it is increasingly more important to ensure the stability and durability of supplies from renewable energy sources (Bettinger et al., 2009). Considering the limited use of traditional renewable sources such as water, sun or wind in the Czech Republic, forests play an important role as a source of renewable energy and renewable material. To ensure a sustainable timber harvest should be an equally important goal as to ensure the stability of forests in changing climatic conditions. For these purposes, modern forest management techniques use different timber harvesting indicators.

Currently, there are two timber harvesting indicators implemented in Czech legislation to express the maximum possible final cut. These are the so-called cutting percentage (CP) (Marušák, Yoshimoto, 2007) and the normal harvest area (Bettinger et al., 2009). Both these indicators come from a regulated forest (Bettinger et al., 2009). However, a regulated forest with a balanced and regulated age class distribution is not only unachievable at present, but also undesirable for forest stability (Priesol, Polák,

1991). In addition, both these indicators are static, which means they only allow planning for one coming decade, without the option of taking into account harvesting possibilities over a longer time horizon. This results in strongly uneven decadal harvests for the whole forest management unit.

Alternative options for determining the optimal volume of harvested timber are represented by so-called operations research (Torres-Rojo, Brodie, 1990; Bettinger et al., 2009). One of these is integer programming (IP), which belongs to the classical mathematical programming methods. The second one is a heuristic method, simulated annealing (SA). This method should allow finding a local maximum of the objective function that differs only slightly from the value of the global maximum, but requires an incomparably shorter computation time.

Linear programming methods, to which integer programming also belongs, are widely used today for optimization in many fields, and are a keystone of operations research (Hillier, Lieberman, 2010). A special case of integer programming is a case where all variables are binary, meaning their possible values are either 0 or 1. Resolving problems by integer

\* Supported by the Technology Agency of the Czech Republic, Project No. TA01020832, and by IGA (Internal Grant Agency of the Faculty of Forestry and Wood Sciences of CULS Prague, Project No. 20124331.

programming is often computationally demanding, and can even take several hours (Jablonský, 2007).

When encountering complicated problems, the mathematical methods of linear or integer programming often fail altogether, or require too much time to find the optimum solution (Li et al., 2010). In such a case, it is not required to find the optimum solution, but any acceptable solution that approaches the optimum solution as closely as possible and requires much less computation time. For this, heuristic methods are usually used, simulated annealing being one of them (Hillier, Lieberman, 2010).

In many optimization problems the objective function has several local extremes and just one global extreme. For example, the traditional climbing algorithm stops when the first extreme is reached. In more complicated cases, the result obtained by this method simply cannot be right. It is therefore necessary to find an appropriate way to leave a local minimum. Sometimes, this means accepting a partial solution that is “worse” than the preceding one (Mehlhorn, Sanders, 2008). Simulated annealing is one of the methods that comply with this assumption. Bertsimas, Tsitsiklis (1993) stated that “simulated annealing is a probability method, designed by Kirkpatrick, Gelett and Vecchi (1983) and Cerny (1985) to find the global minimum of a function that can have several local minima”.

When resolving problems using SA algorithms, it is important to make a cooling plan (Bertsimas, Tsitsiklis, 1993). Its parameters are the initial temperature ( $T_0$ ) and the cooling function  $T_{z+1} = f(T_z)$ . The cooling function can be either very complicated or simpler with a coefficient that at each step lowers the temperature by a fixed (relative or absolute) value (Li et al., 2010). Equally important is the final temperature, where the calculation is supposed to stop. For the SA algorithm itself, also the number of iterations is needed, which sets the number of repetitions for each given temperature value (Kangas et al., 2008).

The use of mathematical programming for harvest planning has been discussed worldwide for more than forty years (Johnson, Scheurmann, 1977; Buongiorno, Gilles 2003). Heuristic methods found their place later, mainly as computers developed. The application of simulated annealing to forest management is found, for example, in the work of Lockwood, Moore (1992).

None of the methods described above are used for assessment of final cuts in the real conditions of forest enterprises in the Czech Republic. This paper compares two selected methods and their use for harvest optimization in clear-cutting system and assesses their use for a real forest enterprise in the Czech Republic. The results from the application of these methods to a real forest enterprise and their comparison with the results of the harvest amounts calculated by the cutting percentage indicator method are presented.

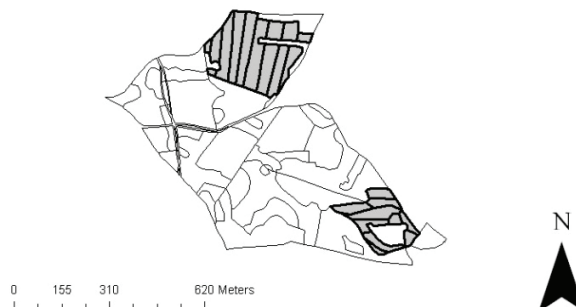


Fig. 1. Suggested regeneration units for the 2nd part of FMA (forest management area)

## MATERIAL AND METHODS

### Data

Data from a real forest management area (FMA) were used for the application of the mathematical model and its solution methods. This FMA consists of three individual, spatially separated parts that will be referred to in this paper as the 1<sup>st</sup> part of the FMA, the 2<sup>nd</sup> part of the FMA, and the 3<sup>rd</sup> part of the FMA. All data were used with the agreement of the FMA owner, but to comply with the rules for protection of personal data, it is not identified more specifically. The final cut was planned for this model FMA for the next 30 years, i.e. for three decades. This corresponds with the traditional time scale when planning final cuts in current forest management plans.

Real data on Norway spruce (standing volume) were used. To predict the growing stock, growth model Czech yield tables were used (Černý et al., 1996).

In the next step, maps from the forest management plan were digitized to *.shp* files and then analyzed in the ArcMap geographical information system. All parts of the FMA that are, or will be in next 30 years, of cutting age, were chosen. These parts of the FMA were then divided into potential cutting units by the editing tools in ArcMap. When editing these units, wind direction, slope, and existing logging roads, on the one hand, were taken into account. On the other, it was important to consider also the legislative parameters for clear-cuts. This means primarily the maximum width, which equals two mean heights of the surrounding stand, and the maximum area of a clear-cut. The cutting units for the 2<sup>nd</sup> part of the FMA are presented in Fig. 1 for illustrative purposes.

The legal requirements for placing clear-cuts were met by an adjacency matrix (Yoshimoto, Brodie, 1994; Matoušek, Nešetřil, 2002). To determine the adjacent cutting units for cutting unit *i*, the definition of Moore’s neighbourhood adjacency was used (Konoshima et al., 2011).

## Mathematical model of harvest optimization

The mathematical model has several parts. The first one is the objective function (1)

$$\max V = \sum_{i=1}^n \sum_{p=1}^3 V_{ip} \cdot x_{ip} \quad (1)$$

where:

$V$  = total amount of final cut over three decades

$V_{ip}$  = standing volume in cutting unit  $i$  in period  $p$

$n$  = total number of potential cutting units in the FMA

$x_{ip}$  = control variable

The control variable is defined by

$$x_{ip} = \begin{cases} 1 & \text{if the unit } i \text{ will be cut in period } p \\ 0 & \text{in other cases} \end{cases} \quad (2)$$

Because this is a unit restricted model (URM) (Crowe et al., 2003), every binary variable in the model represents specifically one proposed cutting unit designed for cutting at the time of elaboration of the plan.

The other parts of the mathematical model for harvest optimization are the constraints. One of these is that each unit can be cut just once per planned period, which can be generalized as

$$\sum_{p=1}^3 x_{ip} \leq 1 \quad \forall i \in N \quad (3)$$

where:

$i$  = ordering number of unit ( $1 - n$ )

$N$  = set of all  $i$  of  $n$  elements

Marušák (2007) states equality for these conditions. That would mean that the unit has to be cut at some time during the planning horizon. At the same time, however, his models are computed for five decades, whereas the present paper uses calculations for just three decades, which is the most frequently used cutting period in the Czech Republic. For this reason, the inequality has been chosen for these constraints so that they are not too rigorous for the complete model.

Conditions that originate in the spatial relations between the cutting units can be set down using analytic algorithm (Yoshimoto, Brodie, 1994), e.g.:

$$\mathbf{M} \cdot \mathbf{X} \leq \mathbf{A} \mathbf{I}, \text{ where} \quad (4)$$

$$\mathbf{M} = \mathbf{A} + \mathbf{B} \quad (5)$$

where:

$\mathbf{A}$  = adjacency matrix

$\mathbf{B}$  = diagonal matrix in which the  $i^{\text{th}}$  diagonal element  $b_{ii}$  is defined by  $b_{ii} = A_i I$  ( $A_i = i^{\text{th}}$  row vector of adjacency matrix  $\mathbf{A}$ )

$\mathbf{M}$  = modified adjacency matrix

$\mathbf{X}$  = control vector for control variables  $x_p$

$\mathbf{I}$  = unit vector ( $n \times 1$ )

The last constraint is the condition of harvest balance across the planning horizon:

$$(1-\alpha)V_{p-1} \leq V_p \leq (1+\alpha)V_{p-1} \quad (6)$$

where:

$\alpha$  = percentage harvest difference between sequential periods

$V_p$  = total harvest in period  $p$

For the calculations using integer programming, we used the optimization software Lingo (Version 11.0.1.4, 2008). For the simulated annealing calculations, we designed our own software.

Three alternative variants were considered for each method. The variants for integer programming method differed in the allowed maximum relative difference between harvests in different decades, and were 5% (IP5), 10% (IP10) and 15% (IP15). The same variants for the simulated annealing method differed in the maximum relative difference between harvests in different decades were calculated as well: 5% (SA5), 10% (SA10), and 15% (SA15). For the simulated annealing calculations, the input parameters, i.e. the cooling plan, had to be derived first. At first, the values from Boston, Bettinger (1998) were used. The initial "temperature" values thus were 500 000, 900 000, and 1 500 000. The final "temperature" value was set to 5 for all options. The numbers of repetitions for one temperature value were 100, 200, and 300, and the reduction factors were set to 0.950, 0.975, and 0.999. Each combination of these factors was run five times and the results averaged. The success of the SA variants is expressed as the ratio of the SA variant with the corresponding IP variant, multiplied by 100:

$$success(\%) = \frac{SA}{IP} \cdot 100 \quad (7)$$

## RESULTS

The harvest volume for the first to third periods (H1, H2, H3) and the total harvest volume (H) obtained by the IP alternative are presented in Table 1. The harvest volumes (H1, H2, H3) and total harvest volume (H) obtained by the SA alternatives are presented in Table 2.

For SA, the initial parameters ( $T_0$  = initial temperature,  $n$  = iteration number) from Boston, Bettinger (1998) were used. The resulting calculated values for each combination are shown in Table 3.

These results, however, are not very successful – the success defined above was less than 93% while has reached more than 98% in the literature (Boston, Bettinger, 1998; Crowe, Nelson, 2005).

It is necessary to consider not only the total number of iterations, but also the true meaning of the individual

Table 1. Results of integer programming method

	IP5 (5% difference)	IP10 (10% difference)	IP15 (15% difference)
H1 (m <sup>3</sup> u.b.)	13 248	12 882	12 505
H2 (m <sup>3</sup> u.b.)	13 902	14 126	14 347
H3 (m <sup>3</sup> u.b.)	13 910	14 170	14 379
H (m <sup>3</sup> u.b.)	41 060	41 178	41 231
Time (min : sec)	2 : 41	1 : 52	1 : 40

H = total harvest volume , H1–H3 = harvest volume for the first to third periods, u.b. = under bark

Table 2. Results from the simulated annealing method

	SA5 (5% difference)	SA10 (10% difference)	SA15 (15% difference)
H1 (m <sup>3</sup> u.b.)	12 739	12 543	12 275
H2 (m <sup>3</sup> u.b.)	13 363	13 667	13 905
H3 (m <sup>3</sup> u.b.)	13 372	13 785	13 953
H (m <sup>3</sup> u.b.)	39 474	39 995	40 133
Time (min : sec)	23 : 14	22 : 56	22 : 55
Success (%)	96.1	97.1	97.4

H = total harvest volume , H1–H3 = harvest volume for the first to third periods, u.b. = under bark

Table 3. Values of objective function for different combinations of input parameters  $T_0$  and  $n = 100, 200, 300$ 

	Values of reduction factors		
	0.950	0.975	0.999
$T_0 = 500\ 000$			
$n = 100$	32 405	33 599	37 633
$n = 200$	33 518	35 031	38 958
$n = 300$	33 002	34 092	39 196
$T_0 = 900\ 000$			
$n = 100$	32 549	31 774	37 806
$n = 200$	32 755	34 570	38 197
$n = 300$	33 598	35 227	38 578
$T_0 = 1\ 500\ 000$			
$n = 100$	31 941	32 876	37 976
$n = 200$	33 327	33 296	38 495
$n = 300$	33 464	34 900	38 491

$T_0$  = initial temperature,  $n$  = iteration number

parameters in the simulated annealing algorithm. The total number of iterations of an SA algorithm is dependent on the range of “temperatures”, initial and final: the bigger the range is, the greater will be the success of the solution. The other two parameters, i.e. the number of iterations for a given temperature value and the reduction factor, influence the total number of solutions and have also other consequences that need to be considered when determining their input values.

The reduction factor influences the probability of accepting a wrong solution. This means that for values approaching 1, the probability of accepting an inappropriate solution rises, especially at the beginning of the calculation.

The number of iterations for a given temperature value also plays an important role in the proposed algorithm. The algorithm, as it is, will move one step further even in the case that the randomly chosen unit does not comply with the constraints. The stricter these conditions are, the lower is this value and the greater is the probability that not even one random choice will meet the constraints. In other words, the calculation moves one step further without changing the objective function. This is why we decided to change the initial values of the iteration numbers for given temperature values. These new parameters and results are shown in Table 4. As before, the given results are averages from five runs.

Table 4. Values of objective function for different combinations of input parameters  $T_0$  and  $n = 600, 1\ 200, 1\ 500$

	Values of reduction factors		
$T_0 = 500\ 000$	0.950	0.975	0.999
$n = 600$	35 531	36 674	38 791
$n = 1\ 200$	36 279	36 567	39 698
$n = 1\ 500$	36 413	37 935	39 396
$T_0 = 900\ 000$	0.950	0.975	0.999
$n = 600$	35 802	36 734	38 845
$n = 1\ 200$	36 184	37 218	39 232
$n = 1\ 500$	36 280	37 069	39 561
$T_0 = 1\ 500\ 000$	0.950	0.975	0.999
$n = 600$	34 954	35 908	39 147
$n = 1\ 200$	36 085	37 216	39 631
$n = 1\ 500$	35 522	36 626	39 717

$T_0$  = initial temperature,  $n$  = iteration number

We obtained better results with these parameter values, consequently for the final calculation, the combination with the best result from these was chosen: initial temperature 1 500 000, number of iterations for a given temperature value 1500, and reduction factor 0.999. Again we ran five series of calculations for each variant of the allowed harvest volume difference between periods (5, 10, and 15%). From each series, the solution with the highest obtained value was chosen. Table 2 then shows these results obtained by the simulated annealing method for input parameters for the given variants (SA5, SA10, and SA15). This table also gives the percentage of success of each solution, calculated as the relative fraction of the results compared to the corresponding result obtained by the integer programming method.

Fig. 2 illustrates the process of finding the global maximum of the function by the simulated annealing method. It shows several local maxima that were ignored by the algorithm during the computation, which continued until it reached the extreme of the function that approaches the global maximum.

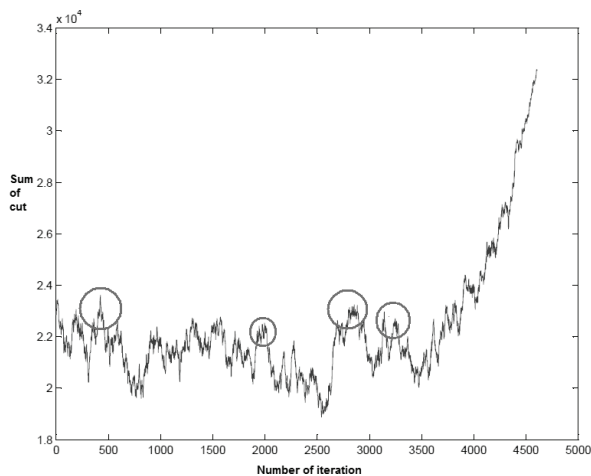


Fig. 2. Changes of the objective function with changing number

Fig. 3 shows the absolute differences in the predicted final cuts for the model FMA proposed by the cutting percentage method and the individual IP and SA variants. The values were taken from the forest management plan of the model FMA. An important difference is found in the decrease of proposed harvested volumes by the cutting percentage method for the 2<sup>nd</sup> decade. This decrease reaches approximately 30% compared to the 1<sup>st</sup> decade.

The results of the different variants of the simulated annealing algorithm do not show any clear relationship between the priority of the chosen units and the identity of the model. The graphical results then clearly show that this method follows no rules when choosing cutting units and does a random selection.

## DISCUSSION

Better results were achieved by using the integer programming method (IP5, IP10, and IP15) than by using the simulated annealing method calculations

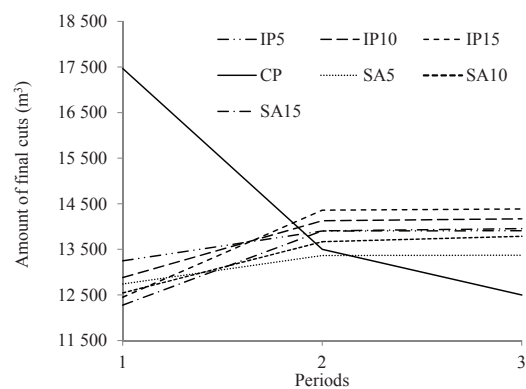


Fig. 3. Graphical comparison of absolute values of harvest volumes suggested by different methods

(SA5, SA10, and SA15). Also all the results confirm that increasing the rigorousness of the restricting conditions increases the time needed to reach a successful solution of the model. The most time-demanding models were IP5 and SA5. At the same time, higher values of the whole harvested volume were obtained in models with less restrictive conditions (IP15 and SA15). This is caused by the lower standing volumes in the individual units in the first period. If the harvested volume in the individual periods should be kept approximately equal (IP5 and SA5) and also comply with all other conditions, then the harvested volume in the 2<sup>nd</sup> and 3<sup>rd</sup> periods has to adjust itself according to the volume of the 1<sup>st</sup> period. These results agree with the total number of stand units chosen in different variants, which is the lowest in alternatives with more restrictive conditions. This example gives us an insight into the optimum solution; it does not necessarily employ the highest number of chosen units, but it finds the units that have the highest standing volumes.

With a high total number of iterations, it is obvious that some units (those significantly increasing the objective function) will be drawn repeatedly in different variants. On the other hand, there are units that were not once chosen for cutting in any of the six alternative solutions, and some units that were chosen just in one alternative. To choose these units would not be expected to increase the value of the objective function, but it would probably decrease it due to the adjacency conditions. It can be expected that if the planning time horizon were longer than three decades, these units would be included in the calculation as well.

However, there are clear differences in the choice of stand units between individual alternatives of calculation. The calculated volume of harvested timber for the 1<sup>st</sup> part of the FMA does not differ between IP5 and IP10, only IP15 is different. Due to this change, the calculation had to change all the units in the IP15 variant so that the adjacency conditions were still met. The same pattern of differences can be observed for the 2<sup>nd</sup> part of the FMA (IP15 differs from IP5 and IP10), but the IP15 variant has the exact opposite distribution of harvesting in the 2<sup>nd</sup> and 3<sup>rd</sup> periods than IP5 and IP10. The last part of the FMA shows differences among all three alternatives, which is caused simply by the fact that more choices allow more combinations. One variant of harvest fluctuation is mainly used (Bašek, Keleş, 2005). More variants, as presented in this paper, enable us their comparison and selection of the better variant according to the spatial distribution of the cutting units.

When looking at the results of simulated annealing, a similar description of the differences between individual calculation variants (SA5, SA10, and SA15) is more difficult. This is because all the results confirm the random selection of stand units for cutting. All three variants therefore show different results for all three parts of the forest enterprise. But even so, it is an interesting result that for the 2<sup>nd</sup> part of the FMA,

the harvest distribution between the 2<sup>nd</sup> and 3<sup>rd</sup> periods is exactly the same as for IP5, IP10, and IP15, even if SA15 has its units divided in the opposite order than all other variants. This is again caused by the possible number of potentially adjacent units: when this number decreases, the number of possible combinations also decreases. Analogously to the previous paragraph, a selection of the better variant according to spatial distribution can be done as well. This would benefit the forest manager and it is not possible when only one variant is computed (Boston, Bettinger, 1998).

Current methods do not consider balancing the harvest between periods and there is no way to incorporate conditions into those models to ensure it. In contrast to the cutting percentage methods, both methods used in this work comply with harvest balance conditions in all considered variants. The operations research methods will help us make a better decision for the final cut prescription in the next planning period (Marušák, 2007).

The discussion of final computation times for both methods is of equal importance, mainly because the obtained values are in complete contrast with the results in other papers (Boston, Bettinger, 1998). The computation times needed for one calculation using the simulated annealing method should have been in the order of minutes (Boston, Bettinger, 1998) or tens of minutes (Crowe, Nelson, 2005), but calculations using integer programming should have had the opposite results. However, there are many explanations for these obtained values, for example the possibilities of computational techniques are constantly increasing and the same goes for the computing power of personal computers. In combination with professional optimization software, it is no problem to get results very fast. Moreover, the optimization model used in this work had neither many restricting conditions, nor many variables. Other authors have searched for the optimal solution when there are more variables (Lockwood, Moore, 1992; Crowe, Nelson, 2005). On the other hand, the programme used for the calculations using simulated annealing had not been optimized for computing time. We can assume that for more planning periods, time needed for the calculation will be longer.

## CONCLUSION

The volume of the final cuts, when calculated by currently used harvesting indicators, reflects only the current situation (i.e. area or volume) of the cutting age classes. No information about younger age classes or the influence of potential planned final cuts on the evolution of the age structure of the forest enterprise is taken into account in those optimization models. Current harvesting indicators come from the regulated forest, which may be suitable for large areas, like the

whole country, because there, the regulated age class distribution could be expected. However, for small forest enterprises, there is a growing probability that the age structure is unbalanced and harvesting indicators therefore lose their validity.

For a wider application in practical forest management planning of the methods and practices described in this paper, more research investigating various models, differentiated by sets of stands, natural conditions, owner's requirements and other factors, is undoubtedly necessary. In the case we want to continue to apply the SA method for final cut optimization, we will need to focus on optimizing the programming code of the software designed for this research. Even so, according to the first achieved results, the use of these methods for spatial and temporal optimization of harvest planning not only appears to be acceptable, it also seems that in the context of the forests of the Czech Republic, it is even more fitting than the use of the classical harvesting indicators. The results show that in the case of small forest management areas in the Czech Republic, the use of heuristic methods is less suitable than exact optimization methods such as IP.

## REFERENCES

- Başkent EZ, Keleş S (2005): Developing alternative wood harvesting strategies with linear programming in preparing forest management plans. *Turkish Journal of Agriculture and Forestry*, 30, 67–79.
- Bertsimas D, Tsitsiklis J (1993): Simulated annealing. *Statistical Science*, 8, 10–16.
- Bettinger P, Boston K, Siry JP, Grebner DL (2009): *Forest management and planning*. 1<sup>st</sup> Ed. Academic Press, New York.
- Boston K, Bettinger P (1998): An analysis of Monte Carlo integer programming, simulated annealing and tabu search heuristics for solving spatial harvest scheduling problems. *Forest Science*, 45, 292–301.
- Buongiorno J, Gilles JK (2003): *Decision Methods for Forest Resource Management*. 1<sup>st</sup> Ed. Academic Press, San Diego.
- Černý M, Pařez J, Malík Z (1996): Yields and enumeration tables for the main tree species (spruce, pine, beech, oak). The Institute for Forest Ecosystem Research, Jílové u Prahy. (in Czech)
- Crowe KA, Nelson JD (2005): An evaluation of the simulated annealing algorithm for solving the area-restricted harvest-scheduling model against optimal benchmarks. *Canadian Journal of Forest Research*, 35, 2500–2509.
- Crowe KA, Nelson JD, Boyland M (2003): Solving the area-restricted harvest scheduling model using the branch and bound algorithm. *Canadian Journal of Forest Resource*, 33, 1804–1814.
- Hillier FS, Lieberman GJ (2010): *Introduction to operations research*. 9<sup>th</sup> Ed. McGraw-Hill, New York.
- Jablonský J (2007): *Operation research – quantitative models for economic decision-making*. 3<sup>rd</sup> Ed. Professional Publishing, Prague. (in Czech)
- Johnson KN, Scheurmann HL (1977): Techniques for prescribing optimal timber and investment under different objectives – discussion and synthesis. *Forest Science, Monograph* 18.
- Kangas A, Kangas J, Kurttila M (2008): *Decision support for forest management*. 1<sup>st</sup> Ed. Springer-Verlag, Berlin.
- Konoshima M, Marušák R, Yoshimoto A (2011): Spatially constraints harvest scheduling for strip allocation under Moore and Neumann neighbourhood adjacency. *Journal of Forest Science*, 57, 70–77.
- Li R, Bettinger P, Boston K (2010): Informed development of meta heuristic for spatial forest planning problems. *The Open Operational Research Journal*, 4, 1–11.
- Lockwood C, Moore T (1992): Harvest scheduling with spatial constraints: A simulated annealing approach. *Canadian Journal of Forest Research*, 23, 468–478.
- Marušák R (2007): Alternative harvest scheduling for final cut with respect to silvicultural requirements. *Lesnícky časopis – Forestry Journal*, 53, 117–127.
- Marušák R, Yoshimoto A (2007): Comparative analysis on cutting possibilities derived from different allowable cut indicators in Slovakia. *Formath Kobe*, 7, 223–238.
- Matoušek J, Nešetřil J (2002): *Chapters from discrete mathematics*. 1<sup>st</sup> Ed. Nakladatelství Karolinum, Prague. (in Czech)
- Mehlhorn K, Sanders P (2008): *Algorithms and data structure. The basic toolbox*. 1<sup>st</sup> Ed. Springer, Berlin – Heidelberg.
- Priesol A, Polák L (1991): *Forest management*. 1<sup>st</sup> Ed. Příroda, Bratislava. (in Slovak)
- Torres-Rojo JM, Brodie JD (1990): Adjacency constraints in harvest scheduling: An aggregation heuristic. *Canadian Journal of Forest Research*, 20, 7, 978–986.
- Yoshimoto A, Brodie JD (1994): Comparative analysis of algorithms to generate adjacency constraints. *Canadian Journal of Forest Research*, 24, 1277–1288.

Received for publication on January 23, 2013

Accepted for publication on April 24, 2013

---

### Corresponding Author:

Ing. Jan Kašpar, Czech University of Life Sciences Prague, Faculty of Forestry and Wood Sciences, Department of Forest Management, Kamýcká 129, 165 21 Prague 6-Suchbát, Czech Republic, phone: +420 224 383 796, e-mail: kasparj@fd.czu.cz

---