MECHANICAL PROPERTIES OF GRANULAR MATERIALS AND THEIR IMPACT ON LOAD DISTRIBUTION IN SILO: A REVIEW

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Mechanical properties of granular materials and their impact on load distribution in storage silo were discussed with special focus on materials of biological origin. Granular materials classification was briefly outlined. The evolution of constitutive models of granular materials developed in the frame of mechanics of continuum was addressed. Analytical methods, Finite Element Methods (FEM), and Discrete Element Methods (DEM) of estimation of silo pressure were discussed. Special attention was paid to the following issues: dynamic pressure switch in the first moment of silo discharge, asymmetry of loads due to eccentric discharge, and impact of uncontrolled increase of moisture content of grain on silo pressures.

granular material; cereal grain; silo; pressure distribution; DEM



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INTRODUCTION

The term granular material includes a large group of materials of particular physical, chemical, and mechanical properties. Granular material can be characterized as a three-phase system composed of grains creating a skeleton and a liquid or gases which fill voids. Grain size may range from several meters to several microns. Granular materials can be found across a wide range of scales: in kitchen (sugar, salt), geophysics (sand, gravel), as well as for example in astrophysics (asteroids).

A specific behaviour of granular media which exhibit some properties of gases, liquids, and solids is in the background of considerations that granular materials are the fourth state of matter (Jaeger et al., 1996). That opinion is supported by the following properties of the material: existence of static friction, inelastic collisions, and a very small energy of thermal motions in comparison to potential energy of gravitational field. Granular materials behave like a liquid or like a solid dependently on parameters such as the density of the system, moisture, etc. As compared to a liquid, granular material shows three distinct differences in mechanical behaviour: existence of internal friction, shear strength dependent on the mean stress and independent on velocity of deformation, cohesion that allows to maintain shape enforced under load (for example ratholes or channels).

Storage, handling, and processing of granular materials are procedures required in numerous industries and are of interest to various branches of science and technology such as physics, chemistry, mechanics, agriculture, and engineering. Agriculture and food industry are, next to chemical, power, and pharmaceutical industries, the largest producers and users of granular materials. The equipment for storage and processing of granular materials should meet two basic conditions: predictable and safe operations and high quality of finished products. Due to globalization processes the industrial companies handling particulate materials have been under severe pressure to reduce costs while enhancing the quality of their products.

Granular materials of biological origin are distinguished by large deformability of particles and strong dependence of their mechanical properties on moisture content. Contrary to materials of mineral origin, moisture penetrates inside grain, leading in some cases to qualitative changes in its physical properties. These specific behaviour of granular materials of biological origin need to be considered when adjusting material models, experimental techniques, and technological solutions. The most important is that stored grain is a respiring biological material subjected to microbiological activity. For high-quality preservation during storage, a multidisciplinary approach based on knowledge from several fields(biology, chemistry, toxicology, engineering, and mathematical modelling) is necessary to study the complex interactions among physical, chemical, and biological variables in a stored-grain ecosystem (Jian, Jayas, 2012).

The main objective of the paper was to describe the evolution and the current state of knowledge regarding mechanical properties of granular materials with special focus on agro and food biologically based granular materials and their influence on loads in silos. The paper presents a review of experimental studies and numerical modelling of mechanical properties of granular solids. Special attention was paid to the effects typically found in deposits of cereal grains.

Classification of granular materials

Due to the variety of the granular materials and their properties, their defining and description are rather difficult. The properties of granular materials vary within a very broad range, depending on the origin of a material, the processes of production and processing applied, and on external factors and conditions. The classification of materials is based on different physico-chemical, mechanical, and geometrical properties. The parameters which are taken into account for classification of granular materials are: size of particles, shape, density, flowability, abrasiveness, toxicity, etc.

Based on ISO classification (I S O 3535, 1977), the Mechanical Handling Engineers' Association (MHEA), UK divided the granular material according to the dimension of particles (*D*) into ten cathegories, however C h at t o p a d h y a y et al. (1994) proposed the five following groups:

• dust	$D \le 0.42 \text{ mm}$
• grain	$D \le 3.35 \text{ mm}$
• lump	$D \le 40 \text{ mm}$
• clump	$D \leq 200 \text{ mm}$
 block 	D > 200 mm

The classification of granular materials according to bulk density (ρ) is as follows:

• light	$ ho < 600 \ kg \ m^{-3}$
• medium	$600 \text{ kg m}^{-3} < \rho \le 1100 \text{ kg m}^{-3}$
• heavy	$1100 \text{ kg m}^{-3} < \rho \le 2000 \text{ kg m}^{-3}$
 very heavy 	$ ho > 2000 \text{ kg m}^{-3}$

The flowability defined as a motion of particles with reference to neighbouring particles or along surfaces is the next parameter describing granular materials (P e l e g, 1985). It has a huge influence on processes occurring during storage and handling of materials in industry and agriculture. The conventional classifications of materials according to flowability are derived from the classification based on the flow function (*FF*) proposed by J e n i k e (1961). C h a t t o p a d h y a y et al. (1994) extended J e n i k e 's (1961) classification adding two extreme classes:

 fluidlike flooding 	
 very free flowing 	FF > 10
• free flowing	10 > FF > 4
 average flowing 	4 > FF > 2
• poor flowing	2 > FF

sluggish/interlocked

In the case of agricultural and food raw materials and products, apart from the classifications mentioned above, attention should be paid also to a number of additional features, such as:

- possibility of freezing,
- hygroscopicity,
- toxic properties,
- properties conducive to spontaneous combustion,
- explosive properties.

Constitutive models of granular materials

Modelling mechanical properties of granular materials starts from the Mohr-Coulomb criterion of plasticity assuming that shear strength τ is a linear function of normal stress σ

 $\tau = \sigma \tan \varphi + C$

where:

 φ = internal friction angle

C = cohesion

Resistance resulting from internal friction and cohesion is overcome by external forces in the moment of attaining the yield strength. The Drucker-Prager criterion of yielding (Drucker, Prager, 1952) representing surface of cone in the principal stress space enables researchers to avoid several problems occurring during the application of the Mohr-Coulomb plasticity criterion representing a pyramid in the principal stress space (Fig. 1).

Further developement of mechanics of granular materials resulted in many constitutive models describing real properties of the granular system: nonlinearly elastic, viscoelastic, elasto-plastic, viscoplastic, and hypoplastic. Among the more advanced elasto-plastic models having a broader application for granular materials of plant origin, the models of Ghaboussi-Momen and Lade should be mentioned. They were applied by Z h a n g et al. (1994) to describe the stress–strain relationship for wheat grain mass.

The mentioned models assume that total strain ε_{ij} is a sum of elastic ε_{ij}^{e} and plastic strains ε_{ij}^{p} .

$$\mathcal{E}_{ij} = \mathcal{E}_{ij}^e + \mathcal{E}_{ij}^p.$$

The model of Ghaboussi and Momen adopts the Drucker-Prager yield condition and describes phenomena typical for isotropic and kinematic hardening. It describes especially well the anisotropy of material, the hysteresis of stress-strain cycle, and the evolution of hysteresis loop in the course of multiple loadings. The model of Lade presents plastic strain ε_{ij}^p as a sum of plastic strain related with the compaction of material ε_{ij}^c and the plastic strain related to dilation of material ε_{ij}^d :

$$\mathcal{E}_{ij}^{p}=\mathcal{E}_{ij}^{c}+\mathcal{E}_{ij}^{d}.$$

Despite of obtaining fairly good descriptions of the behaviour of the material, the key parameter of the material - the microstructure - was not considered in most of the models. The micropolar model based on Cosserat's theory provides an opportunity to model the microstructure of granular media in the frame of continuum mechanics models. As a consequence of joining two theories, the model takes into account both the continuum and micromechanical approach. The strain of material results from superposition of displacements and rotations of particles. As a consequence the stress and the couple stress resulting from particles displacements and rotations are considered. The micropolar model treats granular body as a continuum of non-deformable grains with the parameter of microstructure – the characteristic length representing mean grain diameter. The Cosserat's model proposed by Mühlhaus, Vardoulakis (1987), which takes into account the constitutive elasto-plastic relationship of Drucker-Prager and isotropic hardening and softening, proved to be a useful tool in the investigation of localized deformation in granular materials like the shear bands formation.

Micromechanical approach – Discrete Element Method

Contrary to the continuum mechanics, the Discrete Element Method (DEM) proposed by C u n d a l, S t r a c k (1979) is based on elementary interactions between the grains. The method consists in a simplified solution of the equation of motion for each grain of the material. The calculation procedure is based on the assumption that during a very short time step Δt acceleration and speed are constant, and the dis-

turbance of motion of a single grain does not reach further than to the nearest neighbours. This is the key assumption of the method that permits the description of nonlinear interactions occurring among a large number of elements without excessive requirements concerning the calculation memory power. In this approach all the forces acting on a given granule are considered – those resulting from gravity, from interactions with neighbouring granules, and those resulting from the boundary conditions. Then, on the basis of Newton's second law of dynamics, the acceleration of the granule is calculated. Integration in time permits the determination of the new velocity and position of each grain of the system.

The deformation of an individual grain is considered to be infinitely small compared to the deformation of the whole medium. Therefore, it is usually assumed that the grains are rigid and their deformation at the contact points is modelled through their overlapping. The displacements in the normal direction, tangential direction, and those resulting from grain rotation are considered separately. Modelling of interactions between grains usually involves viscoelastic or elasto-plastic contact in the normal direction and viscoelastic-frictional contact in the tangential (shear) direction. A new important improvement of the method introduced by I w a s h i t a, O d a (2000) is introducing rolling friction which models the interaction on contact surface as opposite to interactions on contact point (Fig. 2). Accumulation of energy in the contact points of the granules is modelled by elastic interaction, while viscosity, plasticity, and dry friction model the dissipation of energy. The Hertz-Mindlin contact model is commonly used in commercial software (e.g. EDEM, Version 2.3, 2010) to simulate nonlinear contact interactions.

To model elongated particles (cereal grains), the multisphere method making cluster of overlaping spheres connected by a rigid bond is very helpful. The next step in multisphere method was building clusters of particles connected by elastic bond (Bonded Particles Model – BPM) (Potyondy, Cundall, 2004). The micro-properties introduced by the BPM consist





Fig. 1. Plasticity criteria in principal stress space: Mohr-Coulomb, Drucker-Prager, and Lade-Duncan

Fig. 2. Interactions between grains on contact points (C u n d a 1, S tr a c k, 1979) and on contact surfaces (I w a s h i t a, O d a, 2000)

of stiffness and strength parameters for the particles and the bonds. This model reproduces many features of behaviour of solids, including: elasticity, stress-strain response, with ultimate yield, and fracturing. These new macro-properties arise from a relatively simple set of micro-properties: damage is represented explicitly as broken bonds, which corresponds to macroscopic fractures of agglomerate under applied load.

Silo load calculations and modelling

One of the first propositions of mathematical description of the behaviour of granular material in a silo was published over 100 years ago by J a n s s e n (1895) for evaluation of loads exerted by grain on storage silo. The method was based on an analytical solution of numerical equations describing the balance of forces on differential slice of grains filling the silo. The J a n s s e n 's (1895) approach required experimental values of four material parameters: internal friction angle, wall friction angle, lateral to vertical pressure ratio, and bulk density of the material.

Background for the recent development in granular mechanics and technology was established by J e n i k e (1961) in his study "Gravity flow of bulk solids". Although at that time it was clear that most of processing industries dealt with flow of granular materials, J e n i k e (1961) presented the first comprehensive study of the subject. A broad outline of analytical methods available at the end of the 20th century was given by D r e s c h e r (1991).

Since that time numerous questions of granular mechanics have been solved based on mechanics of continuum and exact analytical solutions of differential equations relating load and deformation. Numerous experimental and numerical research studies have been conducted to determine static and dynamic pressures and flow regimes in silos (J e n i k e, 1961; H o l s t et al., 1999; R o b e r t s, W e n s r i c h, 2002).

Janssen's approach is still treated as the standard reference method of calculation of the silo pressures and is recommended by the silo designing codes (Eurocode 1, 2003). This approach underestimates the lateral pressure exerted on the wall while the exponential shape of pressure distribution corresponds exactly to the experimental finding. To overcome this difficulty E u r o c o d e 1 (2003) recommends the use of overpressure factors racing-up Jannsen's pressures to real experimental values. Eurocode 1 (2003) recommends also taking care of the influence of variability of mechanical properties of granular solids on pressure distribution in silo. A very illustrative example of that influence is the change in the wall friction coefficient of cereal grains resulting from a prolonged sliding of grain along a silo wall during discharge. During this sliding contact a cutin, a wax-like substance from the grain seed coat, accumulates on the smooth contact surface. Cutin acts as a lubricant that smooths the contact surface and changes its frictional properties. During the first 20 cycles of fill and discharge of the model silo made of galvanized steel the three-fold decrease in the wall friction coefficient was observed (Molenda et al., 1996).

For calculations of pressures in the case of more complicated boundary conditions or more advanced constitutive models the Finite Element Method (FEM) must be applied. The application of the micropolar approach into the FEM code (Tejchman, Wu, 1993; Tejchman, 1998) allows for the description of effects occurring in complex systems of granular materials of real sizes. The model appeared to be a very useful tool of investigating non-uniform and unstable behaviour of the material, like the localized shear zones in the interior of the granular material (W δ jcik, Tejchman, 2009).

DEM simulations of silo loads. DEM provides new possibilities of deeper insight into the micro-scale behaviour of bulk solids, which are not available with traditional or even modern approach of continuum mechanics where gradients of displacement and stress are extremely high like during flow around inserts (K o b y ł k a, M o l e n d a, 2013). The rapid development of computer calculation techniques permitted the realization of computer simulations of a variety of processes occurring in granular materials, such as: dynamic effects in silos, mixing, segregation, gravitational discharge from silos (Z h a n g et al., 1993; M a s s o n, M a rt i n e z, 2000; K o u et al., 2002; P a ra f i n u k et al., 2013; K o b y ł k a, M o l e n d a, 2014).

DEM simulations generally produce a huge scatter of inter-particle forces which after averaging provide useful information. An example of the horizontal forces acting on a vertical wall in quasi-static assemblies (6000 particles in two dimensions) is presented in



Fig. 3. Wall horizontal force averaged for 10 particle-wall contacts compared to Janssen's (1895) solution

Fig. 3. Analysis of the distribution of horizontal forces averaged for 10 particle-wall contacts indicated a moderately smooth increase in the force with increase in particle bedding depth (S y k u t et al., 2008). The DEM values are considerably larger as compared to Janssen's (1895) solution. Similarly Balevičius et al. (2011) obtained good agreement of lateral pressure distribution vs material depth with experimental data which were significantly larger than J a n s s e n 's (1895) solution and Eurocode 1 (2003) recommendations. González-Montellano et al. (2012) obtained the pressure distribution of particles similar to maize grains along the vertical direction of the wall reaching its maximum at the silo-hopper transition using DEM. Masson, Martinez (2000) reported on the impact of anisotropy of contact orientations on the pressure distribution.

To model properly dynamics of discharge, like a rapid, thin ouflow of particles, the contact model for DEM simulations must be selected very carefully as cereal grains reveal different behaviour depending on moisture content (Wiacek, Molenda, 2011). The mechanical properties of grains are strongly influenced by the moisture content. Wojtkowski et al. (2010) found that an elastoplastic model of Thornton, Ning (1998) was efficient for simulation of the behaviour of dry rapeseed while a viscoelastic model of Kuwabara, Kono (1987) gave closer estimates of experimental data for wet seeds. Parafiniuk et al. (2013) used both models to simulate discharge of dry and wet rapeseeds from a model silo obtaining good agreement with experimental results. In the case of cohesive grains the adhesion forces should be considered, like described by the JKR adhesion model (Johnson et al., 1971). If a material is very wet, formation of liquid bridges between particles must be considered (An and et al., 2009).

The DEM proved to be a useful tool for the investigation of phenomena that cannot be efficiently described using methods of continuum mechanics. It requires further development for better consistency between the simulations and experimental results. DEM simulations can provide good agreement with experimental data if the material parameters are properly chosen and adequate particles interaction models are applied (K u w a b a r a , K o n o , 1987; T h o r n t o n , N i n g , 1998; Anand et al., 2009). Still a strong limitation of the method is the number of particles which can be considered if simulation should be performed within acceptable time.

There are several silo operations related to storing and handling of granular solids requiring special care. Among them a dynamic pressure switch in the first moment of silo discharge, eccentric discharge, and impact of uncontrolled increase of moisture content of cereal grain require special attention. These three examples of peculiar behaviour are described in the next section of the paper. **Dynamic pressure switch**. Initiation of discharge of granular solids from silos leads to very sudden and strong stress redistributions which result in silo wall pressure ramps. Z h a n g et al. (1993) indicated over 40% increase in lateral pressure with its peak within 0.7 s of the discharge time of wheat from a smooth and a corrugated-walled model silo. The transition of pressure waves within granular medium is frequently reported in literature as an inherent element of dynamic process of discharge of bulk materials in industrial applications. Sudden increase in lateral pressure takes place at the opening of discharge gate accompanied by ramp down of vertical pressure. This effect, sometimes termed "dynamic pressure switch", may create severe pulsations of pressure on silo structures.

Propagation of a rarefaction wave was modelled using both FEM and DEM. Wensrich (2003) studied numerically the motion of rarefaction wave in a tall cylindrical silo using one-dimensional version of Janssen's equation and a hypoplastic constitutive model of the material. As the rarefaction wave results in dilation of the material, the decrease in vertical and lateral pressure as well as reduction in density was obtained directly behind the wave front. The exponential growth of the amplitude of the rarefaction wave as it travels up the material bed obtained from numerical simulations was found to correspond very well with experimental data of waves travelling upwards through the mass-flow silo from the transition giving rise to an increase in the amplitude of the dynamic wall pressures (Roberts, Wensrich, 2002; Wensrich, 2002; Roberts, 2012).

Pressure waves as well as associated discontinuity in velocity fields in the granular material during discharge create large pulsations which shake the silo structures (T e j c h m a n, G u d e h u s, 1993; W e n s r i c h, 2002). This discontinuous behaviour results in severe dynamical effects of bin structures, i.e. pressure pulsations (called silo music) and shocks (called silo quake), non-uniform distribution of pressure, and dynamic overpressures. Pulsating gravity flow in silos arises as a result of changes in density during flow and by varying degrees of mobilization of the internal friction and flow channel boundary surface friction (R o b e r t s, W e n s r i c h, 2002; M u i t e et al., 2004; W a n g et al., 2012).

Experimental observations indicate that in the case of plain bin hopper rarefaction wave may take form of localized discontinuity in velocity fields which moves upward the bin (Bransby, Blair-Fish, 1975; Drescher et al., 1978; Rong et al., 1995). The shear bands move upward into the vertical part of the bin. In that case the deformation mechanism is the movement of rigid blocks of material that slide over one another along the rupture surfaces. FE calculations with non-local hypoplasticity (Wójcik, Tejchman, 2009) showed that the shape of internal shear zones depended on the wall roughness, initial solid density, and silo form. The shear zones reflected from smooth hopper walls while in the hopper with very rough walls, shear zones occurred only in the material core. Shear zone appeared also in the vertical part of the silo in the case of a very rough wall. Shear localizations emerge from the self-organization of large numbers of particles with long-range geometrical interactions and rearrangements of groups of particles governed by contact forces between particles (O r d et al., 2007).

Eccentric discharge. In a practice of silo operations sometimes it is convenient to apply an eccentric discharge with the silo bottom outlet located at any eccentricity from the centre of gravity of the silo crosssection. Eccentric discharge has been shown to create strong pressure asymmetry. A lot of studies have led to recommendations for silo design codes (Borcz, Hamdy, 1991; Guaita et al., 2003; Łapko, 2010) (Fig. 4). Such pressure asymmetry causes redistribution of vertical and horizontal forces in the silo and produces bending moments in the cylindrical silo wall crosssections which may lead to ovalization of the wall or create particularly high loads that may result in silo failure. If the outlet eccentricity exceeds the critical value of 0.25, a special procedure of calculation of silo pressure distribution is recommended by Eurocode 1 (2003). The highest pressure asymmetry appears for the discharge orifice located at half the radius of the silo floor. Smoother silo wall resulted in larger asymmetry of pressure distribution.

Asymmetry of silo pressure can be created also by eccentric filling. This asymmetry may result from non-uniform bulk density distribution of the material over the cross-section of silo, non-uniform levelling of material, and also from anisotropy of the bedding of granular material produced by grains rolling along the surface of the cone of natural repose. The potential for eccentric filling to decrease load asymmetry during eccentric discharge was investigated by M o l e n d a et al. (2002). Line A in Fig. 5 represented the resultant moment of force exerted by grain on the silo wall during discharge when filling and discharge gates were located on the same side of the silo. Load asymmetry increased momentarily after opening the discharge gate. Line B represented the wall moment vs time relationship for the test when the filling chute and the discharge gate were located on the opposite sides. For this condition, load asymmetry decreased at the onset of discharge (M o l e n d a et al., 2002). Similar findings were obtained by K o b y ł k a , M o l e n d a (2013) in DEM simulations of eccentric filling and discharge.

Swelling pressure. The uncontrolled increase of moisture content may take place in stored grain due to grain respiration or as a result of wetting with ambient air during aeration. Increased grain moisture content leads to an increase in volume. Walls of the silo confine deformation of the grain in the horizontal direction that may lead to an increased lateral pressure (Blight, 1986; Britton et al., 1993). Dale, Robinson (1954) investigated the effects of moisture content changes on the pressure of grain in bins. They indicated that when the moisture content of grain was increased by 4% w.b.(wet basis), the lateral wall pressure increased as much as six times and the vertical floor pressure increased four times.

Zhang, Britton (1995) developed a theoretical model to estimate the relationship between increased moisture content and increased lateral pressure. The model was based on the assumption that an increase in grain volume was equal to the volume of absorbed water. In the next step of the approximation, a decrease in grain elasticity due to moisture content increase was taken into account (Horabik, Molenda, 2000). When kernels swelled, the contact forces increased. At the same time the modulus of elasticity decreased. As a consequence, the pressure reached its maximum value





Fig. 4. Pressure distribution during off-centre discharge according to E u r o c o d e 1 (2003)

Fig. 5. Moment of force exerted on silo wall for off-centre filling and discharge: filling and discharge on the same side (A) and on the opposite sides (B) (M olenda et al., 2002)



Fig. 6. Mean lateral pressure $\boldsymbol{\sigma}_n$ on the silo wall as influenced by the moisture content increase

and then decreased. Resultant volumetric strain is a sum of two independent components: strain generated by external pressure and strain resulting from grain swelling. The rate of mean lateral pressure increase in the model silo was found to be about 125 kPa kg⁻¹ (Fig. 6).

CONCLUSION

Granular solids constitute a very wide group of materials of specific mechanical, physical, and chemical properties. In that group granular materials of biological origin represent a very important source of products of agriculture and food processing industry. Precisely determined parameters of mechanical properties of granular materials and properly chosen constitutive models are fundamental for proper design and control of storage, handling, and processing of those materials. There have still been a lot of particular and specific operations not fully understood or precisely described yet. A few examples of such processes discussed in the paper indicate that further development of tools for modelling the mechanics of granular solids is necessary. One of the most promising tools is the DEM. All these actions are necessary to ensure that the equipment used for storage and processing of granular materials meets two basic demands: predictable and safe operations and high quality of processed materials.

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