TANGENT CURVE FUNCTION DESCRIPTION OF MECHANICAL BEHAVIOUR OF BULK OILSEEDS: A REVIEW*

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The application of tangent curve mathematical model for description of mechanical behaviour of selected bulk oilseeds, namely jatropha, sunflower, rape, garden pea, and common bean in linear compression was reviewed. Based on the review analysis, the tangent curve function has been developed using MathCAD 14 software which employs the Levenberg-Marquardt algorithm for data fitting optimal for tangent curve approximation. Linear compression parameters including force (N), deformation (mm), energy (J), and/or volume energy (J·m^{-3}) can equally be determined by the tangent model. Additionally, the theoretical dependency between force and deformation characteristic curves can be defined by the force coefficient of mechanical behaviour (N) and deformation coefficient of mechanical behaviour (mm^{-1}) of the tangent model. In conclusion, the review results, however, shows that the tangent curve mathematical model which is dependent on experimental boundary conditions is potentialy useful for theoretical description of mechanical properties and deformation characteristics of bulk oilseeds in axial compression.

mathematical model; MathCAD 14 software; force-deformation characteristic; force coefficient; deformation coefficient

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INTRODUCTION

In-depth knowledge on mechanical behaviour of bulk oilseeds is an important step for optimization of equipment design, handling, and oil extraction technology (Karaj, Müller, 2010). In the literature, mechanical behaviour parameters such as force, deformation, hardness, and energy as well as deformation characteristics of bulk oilseeds of jatropha (Sirisomboon et al., 2007; Kabutey et al., 2012a, b; Karaj et al., 2008; Karaj, Müller, 2010), sunflower (Gupta, Das, 2000), rape (Izli et al., 2009), soybean (Kibar, Ozturk, 2008), pea (Grzegorz, 2007), palm kernels (Kabutey et al., 2012c) and others have been primarily focused on experimental determination instead of mathematical or simulation models description. Based on this limited information, Herák et al. (2010) first reported the tangent curve mathematical model applying MathCAD 14 software which uses the Levenberg-Marquardt algorithm for data fitting optimal for tangent curve approximation. The model has been experimentally verified with bulk oilseeds such as jatropha, sunflower, rape, garden pea, and common bean (Herák et al., 2011a, b) to illustrate the mechanical behaviour and deformation characteristics. In fact, the tangent curve model considers the bulk oilseeds of a particular oilseed crop as a unit where they are inhibited by the stress inside the pressing vessel, seed moisture content, porosity, as well as the friction between the seeds and the pressing vessel walls which thus influence the mechanical behaviour and deformation characteristics of bulk oilseeds during compression process (Herák et al., 2010; Kabutey et al., 2011). From technological point of view, where screw presses are used for oil extraction, the bulk oilseeds are considered more economical in terms of oil recovery efficiency, minimum energy requirement, and quality of oil (Beerens,

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2007; K a b u t e y et al., 2010; K a r a j, M ü l l e r, 2009). Practically, the knowledge of the mechanical behaviour of bulk oilseeds in linear compression based on mathematical models such as tangent curve function provides vital information for identification of oil point of bulk oilseeds in axial pressing and optimization of oil recovery efficiency with minimum energy requirement in non-linear compression involving screw presses (H e r á k et al., 2010; 2013a, b; K a b u t e y et al., 2013).

The primary objective of the present review study was to appraise the reliability of the tangent curve mathematical model and the need for its transformation and development of a more rigorous model for description of mechanical behaviour of bulk oilseeds in linear compression.

Linear compression test

For the study of mechanical behaviour and deformation characteristics of bulk oilseeds of jatropha (*Jatropha curcas* L.), sunflower (*Heliantus annus* L.), rape (*Brassica napus* L.), garden pea (*Pisum sativum* L.), and common bean (*Phaseolus vulgaris* L.) in linear compression test (Fig. 1), a single pressing vessel (diameter 76 mm) (H e r á k et al., 2010, 2011a, b, 2012a, b; K a b u t e y et al., 2011; 2012a, b, c; 2014) and varying vessel diameters ranging between 40 and 100 mm (D i v i š o v á et al., 2014; H e r á k et al., 2013a; K a b u t e y et al., 2013) have been used. Individual deformation characteristics of bulk oilseeds of various pressing height (20, 30, 40, 50, 60, 70, and 80 mm) were examined using the compression device ZDM 50-2313/56/18 with a chart recorder (VEB, Dresden, Germany) of maximum force 100 kN and speed 1 $mm \cdot s^{-1}$. Based on the compression device used, it was

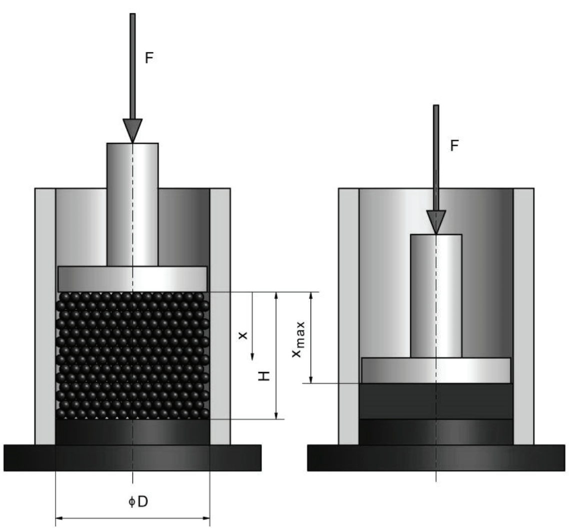

Fig. 1. Scheme of pressing vessel diameter showing the influence of pressing force on measured bulk oilseed pressing seed height (H e r a k et al., 2012a)

indicated clearly that the experimental dependency between the force and deformation characteristics curve required further analysis using the software programme Engauge Digitizer (Version 4.1, 2002) for the determination of the exact compressive force in relation to its corresponding deformation. It is important to note here that the area under the force and deformation curve dependency represents the deformation energy being the energy needed for the deformation of the measured bulk oilseed (G r z e g o r z, 2007; H e r á k et al., 2010) which can be mathematically determined using the equation proposed by H e r á k et al. (2011a, b).

RESULTS AND DISCUSSION

Tangent curve function: a mathematical model

Experimentally, the linear dependency between the force and deformation characteristic curve is shown in Fig. 2 (H e r á k et al., 2011a, b; 2013a; K a b u t e y et al., 2013). This experimental relationship can be described by the function (Eq. 1) (H e r á k et al., 2011a)

$$F(x) = f(x) \tag{1}$$

where:
F = force (N)
x = deformation of bulk oilseed (mm)
$f(x)$ = dependency between force and deformation (Nmm).

The tangent curve function (Eq. 1) was later modified (Eq. 2) (H e r á k et al., 2011b) as:

$$F(x) = A \cdot \lceil \tan(B \cdot x) \rceil^n \tag{2}$$

where:
A = force coefficient of mechanical behaviour (N)
B = deformation coefficient of mechanical behaviour (mm^{-1})
n = exponent of fitted function (-)

Based on Eq. 2, the strain energy and modulus of elasticity in compression can be determined (H e r á k et al., 2011b). Basically, the MathCAD 14 software with Levenberg-Marquardt algorithm for data fitting optimal for tangent curve approximation provides a very simple mathematical algorithm for the tangent curve mathematical model development. This means that the software does not require an advanced computer for data processing unlike simulation programmes such as finite element method (P e t r ů et al., 2012). Most importantly, the tangent curve mathematical model agrees with the experimental boundary conditions. These boundary conditions include the following; when the compressive force increases to infinity the deformation reaches the maximum limit, zero compressive force means zero deformation, and the integral of the tangent curve function is the energy (H e r á k et al., 2010). From Eq. 2, the deformation (mm) can also be described mathematically as shown in Eq. 3 (H e r á k et al., 2012a) as:

$$x = \frac{1}{B}\arctan\left(\sqrt[n]{\frac{F(x)}{A}}\right) \tag{3}$$

Theoretically, when the force is approaching infinity (Eq. 3), then the limit deformation, δ (mm) (Eq. 4) which is dependent on the bulk oilseed height H (mm), can be expressed by the deformation coefficient of mechanical behaviour (H e r á k et al., 2012a, 2013a):

$$\delta = \lim_{F(x)\to\infty} x = \frac{\pi}{2\cdot B} \tag{4}$$

The relationship between compression coefficient G (N) and deformation coefficient of mechanical behaviour B (mm^{-1}) can be expressed (Eq. 5) according to H e r á k et al. 2011a, 2013a as:

$$G = B\cdot H \tag{5}$$

The authors explained that by substituting B from Eq. 5 into Eq. 4, the limit deformation in Eq. 6 could also be expressed as the relationship between the compression coefficient and initial bulk oilseed height as:

$$\delta = \frac{\pi\cdot H}{2\cdot G} \tag{6}$$

where H is the bulk oilseed height (mm) which affects the deformation coefficient of mechanical behaviour, B (mm^{-1}) (H e r á k et al., 2011b, 2013a). The authors again in their studies indicated that using the above mentioned function Eq. 2, an ideal method for regression using the Levenberg-Marquardt algorithm (M a r q u a r d t , 1963) providing numerical solutions to the problem of minimizing deviations in non-linear functions in relation to function parameters can also be attained. Furthermore, the Levenberg-Marquardt algorithm for the determination of the tangent curve function coefficients for fitting linear experimental data with respect to dependency between force and deformation characteristic curves was briefly described by the authors (H e r á k et al., 2011b, 2013a). Most importantly, the Levenberg-Marquardt algorithm is processed in MathCAD software 14 (PTC Software, Needham, USA) (P r i t c h a r d , 1998) which operates with 'genfit' function thus it collaborates with Levenberg-Marquardt algorithm for the development of the tangent curve mathematical model. The tangent curve model as described in Eq. 2 indicates that the linear experimental data that is the dependency between the force and deformation curve, can be fitted based on a single vessel diameter and bulk oilseed height. In terms of the relationship between force and deformation characteristic curves in connection with different vessel diameter and bulk oilseed pressing height, the tangent curve model (Eq. 2) requires modification. This study was subsequently carried-out by the authors Herák et al. (2013a) and Kabutey et al. (2013) on *Jatropha curcas* L. bulk oilseeds. Based on the experimental results obtained, the tangent curve equation (Eq. 2) was mathematically transformed (Eq. 7) as:

$$F(x,D,H) = C\cdot D^2\cdot\left[\tan\left(G\cdot\frac{x}{H}\right)\right]^2 \tag{7}$$

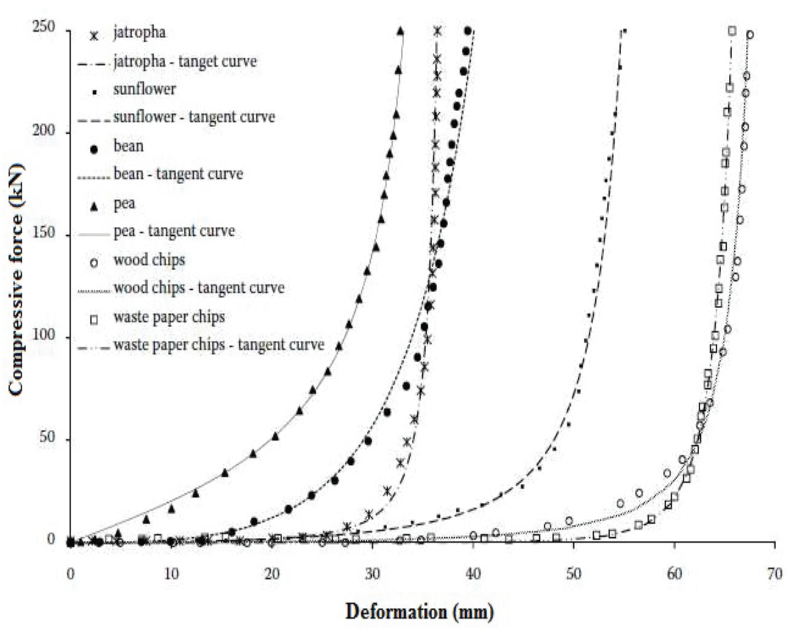

Fig. 2. Experimental data fitted by the tangent curve model (H e r á k et al., 2011a, b)

where:
C = stress coefficient of mechanical behaviour (N·mm^{-2})
D = vessel diameter (mm)
G = compression coefficient (-)
H = bulk oilseed pressing height (mm)

Using Eq. 7, the dependency between force and deformation characteristic curves in relation to varying vessel diameter and bulk oilseed height can be described theoretically.

Tangent curve model coefficients and energy determination

In both practical and theoretical sense, it is necessary to understand the coefficients of the tangent curve model (Eqs 2 and 7). A detailed explanation was published by Herák et al. (2011a, b, 2013a). Briefly, the determined tangent curve model coefficients are valid for specific force between 0 and 100 kN as well as from zero deformation to maximum deformation which can be obtained from the force and deformation characteristic curve dependency. As mentioned earlier, integration of the tangent curve models (Eqs 2 and 7) based on zero deformation to maximum deformation and the fitted curve function (n) value represent the energy (J). In case of jatropha bulk oilseeds, the integral form of Eqs 2 and 7, produced the energy (J) equations (Eq. 8) and (Eq. 9), respectively:

$$\int A\cdot(\tan(B\cdot x))^2\,dx \rightarrow \frac{A\cdot(\tan(B\cdot x) - B\cdot x)}{B} \quad (8)$$

$$\int C\cdot D^2\cdot\left(\tan\left(G\cdot\frac{x}{H}\right)\right)^2 dx \rightarrow \frac{C\cdot D^2\cdot H\cdot\left(\tan\left(\frac{G\cdot x}{H}\right) - \frac{G\cdot x}{H}\right)}{G} \quad (9)$$

The variables in both equations have been defined in the preceding sections. Clearly, this theoretical energy is similar to the experimental energy (Herák et al., 2013a; Kabutey et al., 2013).

Experimental and theoretical description of force-deformation curves

Fig. 2 shows the experimental dependency of force and deformation characteristic curves fitted by the tangent curve function (Eq. 2) (Herák et al., 2011a, b). The fitted function curve (Eq. 7) was also described and reported in the study of Herák et al. (2013a) for *Jatropha curcas* bulk oilseeds. Based on the results of the study, the authors indicated that the tangent curve model described the experimental data which was statistically significant ($P > 0.05$) with a very high coefficient of determination, R^2 using the MathCAD software. Obviously, the authors emphasized that the validity of the tangent curve models (Eqs 2 and 7) was dependent on the force value between 0 and100 kN and also from zero to maximum deformation of particular bulk oilseed pressing height. The explanation is that the force coefficient of mechanical behaviour, A (N) influences the slope of the deformation curves while the deformation coefficient of mechanical behaviour, B (mm) affects the extent of deformation, which is the ratio of the force coefficient of mechanical behaviour to that of deformation coefficient of mechanical behaviour, and practically, this is the initial rigidity of the compression system. Another variable consideration is the exponent function, n (-) which determines the best-fit value of the tangential function for the description of the force and deformation dependency. In the literature (Herák et al., 2011a, b; 2012a, b), different best-fit values have been identified for jatropha, sunflower, rape, garden pea, and common bean bulk oilseeds. These specific values could also vary with variety and maturity stage and moisture content of bulk oilseeds (Kabutey et al., 2011).

Tangent curve model transformation and the development of additional models

In linear compression test of bulk oilseeds, it is relevant to note that both the physical properties of the bulk oilseeds such as moisture content, porosity, variety, and maturity stage could influence the mechanical behaviour and deformation characteristics of the bulk oilseeds (Herák et al., 2010, 2011a, b, 2012a, b; Kabutey et al., 2011; 2012a, b, c; 2014).

The study conducted by Kabutey et al. (2011) revealed that an increase in bulk oilseed moisture content caused a 'serration effect' on the force-deformation curve dependency (Herák et al., 2012a; Kabutey et al., 2014) which increased the deformation value (Grzegorz, 2007). In case of the tangent curve mathematical model, the above mentioned compression factors were constant therefore their varying effect could change the coefficients of the tangent curve model. In addition to the tangent curve model, a new approach that is the reciprocal slope transformation (RST) with the Least Squares Method (LSM) has been useful for description of force-deformation characteristic of jatropha bulk oilseeds in relation to varying pressing heights (Herák et al., 2014). This suggests that apart from tangent curve function, the RST could be applied to explain the mechanical behaviour of bulk oilseeds under compression loading. However, a more rigorous method is required in terms of simulation models such as a finite element method (FEM) which has been already utilized to describe the mechanical behaviour of a single oilseed of jatropha (Petrů et al., 2012) instead of bulk oilseeds which economically meets industrial processing technology. Based on the evaluation results it is theoretically important to build on the tangent curve and the RST mathematical models by using other numerical methods for description of mechanical and deformation characteristics of bulk oilseeds.

CONCLUSION

Based on the review study, the tangent curve mathematical model has been developed using MathCAD 14 software which uses Levenberg-Marquardt algorithm for data fitting optimal for tangent curve approximation. It was observed that the tangent curve model has been theoretically applied not only on bulk oilseeds such as jatropha, sunflower, rape, garden pea, and common bean, but also the description of industrial materials including wood chips and waste paper chips. The exponent function (*n)* of the tangent curve model substantially influences the dependency of the force-deformation curve and also the theoretical deformation energy. The integral of the tangent curve model is energy and this energy model could vary among different bulk oilseeds. A varying effect of compression factors such as seed moisture content could change the tangent curve coefficients. Presently, the tangent curve mathematical model together with Reciprocal Slope Transformation (RST) could be used for the description of mechanical behaviour of bulk oilseeds under compression loading. Numerical methods such as Finite Element Method (FEM) for description of mechanical and deformation characteristics of bulk oilseeds are very necessary. However, it is relevant to combine the presently developed models and future models with the theoretical models such as Darcy, Terzaghi and Hagen Poiseulle (Mrema and McNulty, 1985) to find the relationship between the different coefficients to efficiently describe the oil point and oil flow rate in linear compression environment and thereafter the transformation of the results into the non-linear environment involving mechanical screw presses to minimize energy requirement but maximize oil recovery efficiency.

REFERENCES

Beerens P (2007): Screw-pressing of Jatropha seeds for fueling purposes in less developed countries. Dissertation, Eindhoven University of Technology.

Divišová M, Herák D, Kabutey A, Šleger V, Sigalingging R, Svatoňová T (2014): Deformation curve characteristics of rapeseeds and sunflower seeds under compression loading. Scientia Agriculturae Bohemica, 45(3), 180–186. doi: 10.2478/sab-2014-0106.

Gupta RK, Das SK (2000): Fracture resistance of sunflower seed and kernel to compressive loading. Journal of Food Engineering, 46, 1–8. doi: 10.1016/S0260-8774(00)00061-3.

Grzegorz L (2007): Fracture toughness of pea: Weibull analysis. Journal of Food Engineering, 83, 436–443.

Herák D, Gurdil G, Sedlacek A, Dajbych O, Simanjuntak S (2010): Energy demands for pressing Jatropha curcas L. seeds. Biosystems Engineering, 106, 527–534. doi: 10.1016/j.biosystemseng.2010.06.002.

Herák D, Kabutey A, Sedláček A, Gürdil G (2011a): Tangent curve utilization for description of mechanical behaviour of pressed mixture. Research in Agricultural Engineering, 57, 13–18.

Herák D, Kabutey A, Sedláček A (2011b): Mathematical description of rapeseeds (Brassica napus L.) mixture mechanical behavior under compression loading. Scientia Agriculturae Bohemica, 42, 31–36.

Herák D, Kabutey A, Sedláček A, Gürdil G (2012a): Mechanical behaviour of several layers of selected plant seeds under compression loading. Research in Agricultural Engineering, 58, 24–29.

Herák D, Kabutey A, Divisova M, Svatonova T (2012b): Comparison of the mechanical behaviour of selected oilseeds under compression loading. Notulae Botanicae Horti Agrobotanici Cluj-Napoca, 40, 227–232.

Herák D, Kabutey A, Divisova M, Simanjuntak S (2013a): Mathematical model of mechanical behaviour of Jatropha curcas L. seeds under compression loading. Biosystems Engineering, 114, 279–288. doi: 10.1016/j.biosystemseng.2012.12.007.

Herák D, Kabutey A, Hrabe P (2013b): Oil point determination of Jatropha curcas L. bulk seeds under compression loading. Biosystems Engineering, 116, 470–477. doi.org/10.1016/j.biosystemseng.2013.10.006

Herák D, Blahovec J, Kabutey A (2014): Analysis of the axial pressing of bulk Jatropha curcas L. seeds using reciprocal slope transformation. Biosystems Engineering, 121, 67–76. doi: 10.1016/j.biosystemseng.2014.02.009.

Izli N, Unal H, Sincik M (2009): Physical and mechanical properties of rapeseed at different moisture content. International Agrophysics, 23, 137–145.

Kabutey A, Herak D, Hanus J (2010): Screw press performance for oil extraction from Jatropha curcas L. seeds of different moisture content. Scientia Agriculturae Bohemica, 41, 225–230.

Kabutey A, Herák D, Sedláček A (2011): Behaviour of different moisture content of Jatropha curcas seeds under compression loading. Research in Agricultural Engineering, 2, 72–77.

Kabutey A, Herák D, Chotěborský R, Navrátilová M (2012a): Model for energy and deformation determination of selected oilseeds under compression loading – Short communication. Research in Agricultural Engineering, 58 (4): 155–158.

Kabutey A, Sedláček A, Divišová M, Svatoňová T (2012b): Heat treatment temperature of Jatropha curcas L. seeds under compression loading. Scientia Agriculturae Bohemica, 43, 116–121.

Kabutey A, Divišova M, Sedláček L, W.E. Boatri, Svatoňová T, Sigalingging R (2012c): Mechanical behaviour of oil palm kernels (Elaeis guineensis). Scientia Agriculturae Bohemica, 44, 18–22. doi: 10.7160/sab.2013.440104

Kabutey A, Herak D, Chotěborský R, Dajbych O, Divišova M, Boatri WE (2013): Linear pressing analysis of Jatropha curcas L. seeds using different pressing vessel diameters and seed pressing heights. Biosystems Engineering, 115, 43–49. doi:10.1016/j.biosystemseng.2012.12.016.

Kabutey A, Herák D, Dajbych O, Divišová M, Boatri W. E, Sigalingging R (2014): Deformation energy of Jatropha curcas L seeds under compression loading. Research in Agricultural Engineering, 60, 68–74.

Karaj S, Müller J (2009): Optimization of mechanical extraction of Jatropha curcas L. seeds. Focus Cropping and Machinery, Landtechnik, 64, 164–167.

Karaj S, Müller J (2010): Determination of physical, mechanical and chemical properties of seeds and kernels of Jatropha curcas L. Industrial Crops and Products, 32, 129–138. doi: 10.1016/j.indcrop.2010.04.001.

Karaj S, Huaitalla RM, Müller J (2008): Physical, mechanical and chemical properties of Jatropha curcas L. seeds and kernels. In: Proc. Conference on International Agricultural Research for Development, Stuttgart, Germany, 1–7.

Kibar H, Ozturk T (2008): Physical and mechanical properties of soybean. International Agrophysics, 22, 239–244.

Marquardt DW (1963): An algorithm for the least-squares estimation of nonlinear parameters, SIAM. Journal of Applied Mathematics, 11, 431–441.

Mrema, G. C, Mc Nulty, P. B (1985): Mathematical model of mechanical oil expression from oilseeds. Journal of Agricultural Engineering Research, 31(4), 361–370.

Petrů M, Novák O, Herák D, Simanjuntak S (2012): Finite element method model of the mechanical behaviour of Jatropha curcas L. seed under compression loading. Biosystems Engineering, 111, 412–421. doi: 10.1016/j.biosystemseng.2012.01.008.

Pritchard PJ (1998): Mathcad: A tool for engineering problem solving. McGraw-Hill Science Engineering, New York.

Sirisomboon P, Kitchaiya P, Pholpho T, Mahuttanyavanitch T (2007): Physical and mechanical properties of Jatropha curcas L. fruits, nuts and kernels. Biosystems Engineering, 97, 201–207. doi: 10.1016/j.biosystemseng.2007.02.011.

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